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BPHCL-132
MECHANICS: LABORATORY

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MECHANICS: LABORATORY – INTRODUCTION

In this laboratory course on Mechanics you will be performing various experiments related to basic physics concepts you are learning in the theory course on Mechanics. We have included some experiments on determination of acceleration due to gravity and certain material properties like elastic constants of a wire, spring constant of a spring-mass system. You will also be performing some experiments based on oscillations and waves. Besides this, we have included two units on taking measurements, performing error analysis and plotting the data using proper graphing techniques. Our focus is on training you to master the skill of making precise measurements on fundamental quantities – length, mass and time – and to inculcate the ability to analyze obtained data and understand the physical significance of obtained results.

While taking measurements with different instruments and analysing data you should be aware of the possible sources of error as well as how to correctly record and make calculations taking into consideration the concept of significant figures. So, to give you a feel for these aspects of experimentation, we have discussed the importance of error analysis and method of writing the result with correct number of significant figures in Unit-I. Many a times the analysis of experimental data is simpler if we represent it on proper graphs. Thus in Unit-II we have discussed best practices in graph plotting and making use of appropriate graph formats like linear plots, semi-log plots and log-log plots.

While performing an experiment, it is very important to make right choice of the instruments in order to get best possible results. An important consideration in selecting a particular instrument is its ability to measure with the desired level of precision. The least count of an instrument is one such parameter that plays a vital role in determining the precision of your measurement. To illustrate the importance of this parameter in a measurement we have included length measurement using various instruments like vernier callipers, screw gauge and a travelling microscope in Experiment 1 of this course. In this experiment, you will also be able to apply the corrections for the systematic errors caused during the measurements as explained in Unit-I and arrive at more accurate results

In Experiment 2 you will use fly wheel to obtain the moment of inertia of a rotating body. In the next three experiments you will learn to determine various elastic constants of a material using different techniques. Young's Modulus is one of the most important mechanical properties of solids, particularly for building bridges and erecting columns in high rise buildings. In Experiment 3, you will learn to determine the Young's Modulus of a material by using the method of bending of beams. The depression in the beam can be measured by a microscope as well as an optical lever arrangement. In this experiment, while measuring lengths using optical instruments, you will also learn to remove parallax and take correct measurements. In Experiment 4 you will learn to use Maxwell's needle to determine the modulus of rigidity of a wire. It essentially uses dynamical method where time periods of the oscillating needle are measured under different configurations. Apart from these two moduli, there are two other elastic constants: Bulk modulus and Poisson ratio. In Experiment 5 you will use Searle's apparatus to determine all the four elastic constants.

In the following two experiments (Experiments 6 and 7), you will use the simple harmonic motion performed by a homogeneous mass distributed system (bar pendulum) and asymmetric mass distributed system (Kater's pendulum) to determine the acceleration due to gravity.

Springs have many uses in our daily life. In Experiment 8, you will calculate the spring constant of a spring in two different ways: *Static Method*, where we determine extension as a function of load and *Dynamic Method*, where we measure the period of harmonic oscillations of a spring-mass system.

We all know that life without music would have been less enjoyable. As a student of physics, you would like to know as to how musical instruments like *sitar*, violin, guitar and *ektara* generate music and what factors determine its quality. In Experiment 9 you will study the dependence of frequency of vibrations of a stretched string on applied tension, its mass per unit length and its vibrating length. You will also establish the relation between frequency and wavelength for waves generated on a stretched wire.

In the last experiment (Experiment 10) of this laboratory course you will be studying the Lissajous figures generated by superposition of two mutually perpendicular sinusoidal waves. By studying the shapes of these figures, you will be able to determine the phase relation between the two waves. You will also be able to obtain the frequency relation between two different frequency waves by studying the Lissajous figures generated by them. In this experiment you will have an opportunity to use the Cathode Ray Oscilloscope (CRO), which is a very commonly used electronic instrument in electronics laboratories. You will also get an opportunity to build small electronic circuits to generate waves with phase difference.

The purpose of including these experiments in this laboratory course is to start from a familiar situation and give you experience of planning the experiments with increasing levels of sophistication. The basic purpose of this laboratory course is to inculcate the art of setting up the apparatus, taking measurements, making simple calculations and analysing the results. Moreover, you will appreciate that a lot of good physics can be understood with simple experiments and activities.

We hope that you will have enjoyable experience in the laboratory.

UNIT I

MEASUREMENTS AND ERROR ANALYSIS

Structure

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I.1 INTRODUCTION

As a student of science, you may have done experiments in your school laboratory. You know that many kinds of instruments are used to measure physical quantities. When we measure various physical quantities, it is very important to understand the correct way to make measurements and obtain correct readings. It is also a fact that even the best of the measuring instruments do not yield the true values of the quantities being measured. This is because of their limited accuracy and precision.

We express these measurements as approximate numbers such as 3.2 cm or 3.20 cm. Do you know why we use numbers upto different decimal places and what distinguishes them? While doing computations with these numbers special care is required. For example, the ratio of two measurements such as 32.1/12 is expressed as 2.7 rather than 2.68 or 2.675. Do you know the reason? The number of digits used in a measurement carry some significance regarding the quality of measuring instrument.

In this laboratory course, you will handle various instruments to make measurements. So, it is a good idea to learn some basic concepts related to any measurement. In this unit, you will learn the meaning and usage of measured numbers, with particular reference to precision and accuracy of the experimental result. It is also important to report your measurements in appropriate manner. So in the first two sections (Sec. I.2 and I.3) of this unit, you will learn the meaning of precision, accuracy and about reporting the results in scientific notation with significant digits.

You also know that every measuring device has a least count, which tells us of its ability to measure a physical quantity up to a particular accuracy. It means that the number obtained as a result of (a series of) measurement(s) cannot be said to be 'exact' or 'true'. Further, there can be defects in measuring instruments and even a very careful experimentalist is susceptible to certain personal errors. Both these factors give rise to experimental error.

The uncertainty in any number obtained from a measurement constitutes what is referred to as **error**. It is important to note that within an experiment, the error accumulates in different measurements. Therefore, in Sec. I.4, you will learn about the types and sources of errors. You will also learn how to estimate and possibly eliminate or minimise and account for such errors. In most physics experiments, our objective is to determine the relationship among physical quantities. We carry out calculations using the observed readings in appropriate formulae. In Sec. I.5 we discuss about the propagation of error during such calculations.

In this laboratory course, you will first perform length measurements and then do experiments involving two or more physical quantities.

Expected Learning Outcomes

After studying this unit, you should be able to:

- explain why measurements result in approximate numbers;
- distinguish between precision and accuracy;
- report a measurement in scientific notations with correct number of significant digits;
- identify the sources of error; and
- distinguish between random errors and systematic errors.

I.2 ERRORS IN MEASUREMENTS

Firstly, an error may be caused due to a defect in the measuring instrument itself, such as the **zero error**. Secondly, an error could be due to limitations of human judgement and perception, such as in aligning the end of a rod to be measured with the zero of the scale, or parallax in reading a value. To enable you to better appreciate the inexact nature of measurement, let us consider length measurement. Let us assume that we have a 'perfect' centimetre scale which has clear and equal markings of millimetres. We wish to measure the length of three arrows *A*, *B* and *C* shown in Fig. I.1 using this scale. Let us suppose that we are able to perfectly align the tails of the arrows with zero marking on the scale. (This is impossible to achieve in practice. But let us begin by considering an ideal situation to understand the process of measurement.)

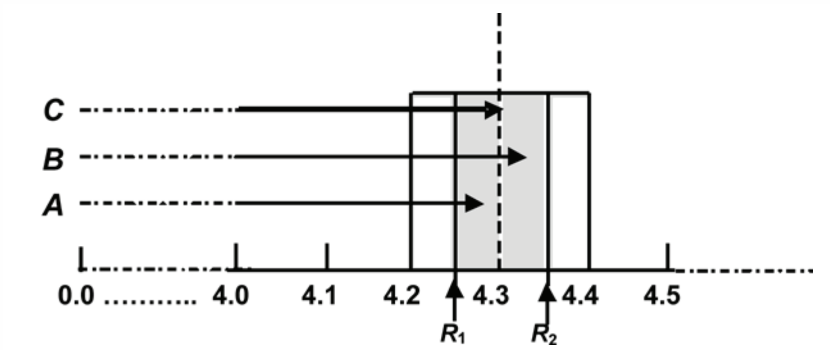


Fig. I.1: The length of all the three (unequal) arrows *A*, *B* and *C* is reported as 4.3 cm. The shaded portion on the scale represents the range of error in this measurement. (The scale is highly magnified.)

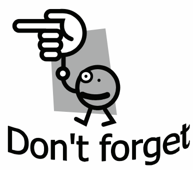
To measure the length of the arrows, we look at the arrow heads. The head of arrow *A* is closer to the 4.3 cm mark than to the 4.2 cm mark. We will report the length of arrow *A* as 4.3 cm to the nearest millimetre. Let us now measure the length of arrow *B*. The head of arrow *B* is closer to 4.3 cm mark than to 4.4 cm mark. Therefore, we will report its length also as 4.3 cm. Similarly the length of arrow *C* would be reported as 4.3 cm. Thus the lengths of all arrows, though different, will be reported as 4.3 cm.

We can conclude that a measurement which is reported as 4.3 cm (which is in the middle of R_1 and R_2) might possibly be in error by 0.05 cm (or one-half of the unit of measure, which is 0.1 cm in this case) or less. It means that in the measurement 4.3 cm, the last digit, 3 is in error. We can generalise this result: **no measurement can ever be exact;** there will always be deviation from the true value due to the limited accuracy of the measuring device/instrument. The inaccuracy is reflected in the last digit.

I.2.1 Probable Error and Precision

We have seen that the maximum error, barring a human error in a measurement, is half of the unit of measurement. The probable (or possible) error is thus due to **inherent imprecision in measuring devices called least count of the instrument**. Measurements having less probable error are more precise. **Since probable error is proportional to the smallest unit of measure the instrument can measure (least count), the instrument having smaller least count gives more precise measurement.** A measurement reported to one-hundredth of a centimetre, such as 5.32 cm is more precise than a measurement reported to one-tenth of a centimetre, such as 5.3 cm.

The *probable error* is half of the unit of measurement.



To be able to determine the precision of any measurement, you may like to attempt an SAQ.

SAQ 1 – Precision in measurement

Consider the following pairs of measurement. Indicate which measurement in each pair is more precise:

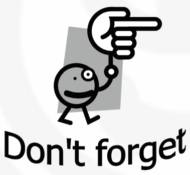
a) 17.9 cm or 19.87 cm

b) 16.5 s or 3.21 s

c) 20.56 °C or 32.22 °C

I.2.2 Relative Error and Accuracy

So far we have considered measurement of nearly equal lengths with emphasis on precision. Let us now consider measurement of far different lengths. Suppose that two measurements yield 3.2 cm and 98.6 cm using the same metre stick. The probable error in both these measurements is equal to 0.05 cm. But the measurement 98.6 cm is bigger than measurement 3.2 cm. Would you say that the 98.6 cm is more accurate? Again, let us consider measurement of time in seconds. How do the measurements 7.4 s and 98 s compare in terms of accuracy? You must have noticed that, the probable error in measuring 7.4 s is 0.05 s, whereas for 98 s, it is 0.5 s. To compare such measurements, we introduce the term ***relative error***, which is defined as the ratio of probable error to the total measurement.



$$\text{Relative Error} = \frac{\text{Probable Error}}{\text{Total Measurement}}$$

In Table I.1, we have calculated relative error in a few typical measurements. The exact method of expressing the relative error will be discussed in section I.5.

Table I.1: Calculation of relative error

Measurement	Unit of measure	Probable error	Relative error
3.2 cm 98.6 cm	0.1 cm 0.1 cm	0.05 cm 0.05 cm	0.02 0.0005
7.4 s 98 s	0.1s 1s	0.05 s 0.5 s	0.007 0.005

Note that in the measurement of 3.2 cm and 98.6 cm, the unit of measure is the same and we say that both measurements are equally precise. But the relative error is less in the larger measurement (0.0005 compared to 0.02) and it is said to be more accurate.

Comparison of measurement 7.4 s and 98 s is more revealing. The measurement 7.4 s is **more precise** than the measurement 98 s (possible errors 0.05 s and 0.5 s, respectively) but **less accurate** (relative error 0.007 as compared to 0.005).

You will therefore appreciate that a **smaller measurement needs to be more precise for the same accuracy**. This is why when measuring the dimensions of a room, metre is used as unit of measurement, while in measuring inter-city distances, the unit kilometre is used for the same accuracy.

Now try to ascertain the accuracy in the given pair of measurements in the following SAQ.

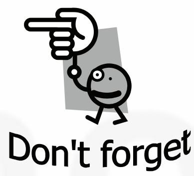
SAQ 2 – **Accuracy in measurement**

Consider the following pairs of measurements. Indicate which measurement in each pair is more accurate:

a) 40.0 cm or 8.0 cm
b) 0.85 m or 0.05 m

Note that in a measurement, errors can be introduced by

- **the measuring instrument due to its inherent imprecision;**
- **limitations of an experimentalist; and**
- **external conditions.**



After understanding the difference between precision and accuracy in measurements based on probable and relative errors, we will now discuss about reporting the observations (readings) taken by you in scientific way.

I.3 REPORTING RESULTS

I.3.1 Scientific Notations

In the scientific notations (SI system of measurement), a measurement is expressed in decimal numerals. You may recall that in inter-atomic distances, very small numbers are obtained, whereas in measuring interstellar distances, we have to deal with very large numbers. In scientific notation, these numbers are written as a number between one and ten using a decimal point notation and then multiplied by an integral power of ten. For example, the diameter of the sun is 1,390,000,000 metre and the diameter of hydrogen atom is only 0.000000000106 metre. In scientific notation, we write the diameter of the sun as 1.39×10^{9} m and the diameter of the hydrogen atom as 1.06×10^{-10} m.

SAQ 3 – **Expressing results in scientific notation**

Express the mass of a water molecule, 0.000 000 000 000 000 000 000 03g, in scientific notation.

You will appreciate that writing numbers in scientific notation makes representation more convenient. Moreover, calculations become easier because we can apply the laws of exponents readily.

I.3.2 Significant Digits

In Sec. I.2.1, you have learnt that a measurement reported as 5.32 cm is more precise than that reported as 5.3 cm. The number of digits in these measurements is three and two, respectively. This suggests that the number of digits used in reporting a measurement have some significance. Whenever you report any measurement, it is important to express it in "correct" number of significant digits. For example, suppose that you are measuring time with a stop watch with least count 0.1 s and reporting readings by taking average of (say) three events. If the time measurements are 5.5s, 5.7s, 6.0s respectively; then the average is 5.733s. Note that these measurements have two significant digits; so you will report the result as 5.7s with two significant digits.

There are certain rules for counting the significant digits. We now state these with some examples.

- All zeros appearing between two non-zero digits are significant. The measurement 107.005 m has six significant digits, whether it is written as 0.107005 km or 10700.5 cm.
- All zeros appearing on the immediate right of a decimal point, i.e. in front of non-zero digits are not significant, when there is no non-zero digit on the left of the decimal. Thus, 0.003 kg has one significant digit, as does 0.7s. However, 0.103 m has three significant digits and so does 0.00783 m.
- All zeros following a non-zero number and to the right of the decimal point are significant. The measurement 47.000 m has five significant digits while 700.000 kg has six significant digits.

You may now like to answer an SAQ.

SAQ 4 – **Significant figures**

Complete the Table I.2 and answer the following questions:

Table I.2

Sl. No.	Measurement (m)	No. of Significant Digits	Unit of Measurement (m)	Probable error (m)	Relative error
1.	0.2	1	0.1	0.05	$\frac{0.05\,\text{m}}{0.2\,\text{m}} = 0.25$
2.	0.20	2	0.01	–	–
3.	0.2000	–	–	–	–
4.	25	–	–	–	–
5.	250	–	–	–	–
6.	25000	–	–	–	–
7.	102	–	–	–	–
8.	1002	–	–	–	–

a) Is there a significance of 'trailing' zeros in the first three measurements?

b) What about the significance of the zeros in the fifth and sixth measurement?

c) What is the significance of zeros between non-zero digits in the seventh and eighth measurements?

After solving SAQ 4, you must have realised that any number is significant, when it affects the relative error. That is the measurement with more number of significant digits have greater accuracy.

Sometimes we take a sequence of whole number measurements such as 32, 30, 28, 26. All these measurements have two non-zero significant digits, except the measurement 30. In such special cases, zero is also significant without any ambiguity.

Now you have understood that errors in measurements can crop up due to various reasons. In the next section you will learn about different types of errors.

I.4 TYPES OF ERRORS

So far you have learnt that errors can arise due to limitations of measuring instruments as they cannot measure smaller than their least count. For instance, a metre scale cannot measure less than 0.1 cm, a vernier callipers measures a minimum length of 0.01 cm and a screw gauge cannot measure distances less than 0.001 cm. Similarly, a thermometer can measure temperature to a precision of half a degree Celsius. When measuring angles, a simple protractor measures to a precision of one degree, but when a vernier is attached to the protractor, as in a spectrometer, we can measure angles more precisely, up to 30″.

In addition to the limitations listed above, which are inherent in the measuring device, other sources of error could be (i) changes in environment, (ii) faulty observation techniques, (iii) malfunctioning of measuring devices, etc.

The errors in any measurement can be classified in two broad categories: *Systematic Errors* and *Random Errors*. Let us now learn about these in detail.

I.4.1 Systematic Errors

Systematic errors arise mostly due to the instruments used in the measurement. They are also called 'determinable' errors and arise due to identifiable causes. For this reason, these can, in principle, be eliminated or corrected. These errors result in measured values being either consistently high or low from the true value. You will typically encounter the systematic errors in the form of following instrumental errors:

- **Zero Error** arises due to wear and tear caused by extensive use. The zero of the vernier scale may not coincide with the zero of the main scale when the jaws are put in contact. The magnitude and nature (positive or negative) of the zero error can be easily determined and corrected. In case of positive zero error, the zero of the vernier scale is on the right of the zero of main scale and opposite is the case for negative zero error. So, for positive zero error, we subtract (and add in case of negative zero error) the value of error from the measured value.

- **Backlash Error** in a screw gauge, a travelling microscope or a spherometer can arise due to wear and tear of a rotating part or defective fitting in the instrument. In this case, a forward or backward rotation may not produce the same result. This is minimised by rotating the screw head of the measuring device in only one direction from the initial to the final point of measurement.
- **End Correction** arises when the edge (zero marking) of a scale is not distinctly visible due to wear out. This leads to an error if one tries to keep the zero of the scale at the starting point. This can be eliminated easily by shifting the reference point of the scale to a definite and distinct point (say, 1 cm mark).
- **Errors due to changes in a physical quantity** can take place during the course of the experiment. For example, the resistance in electrical circuits can change due to the heat generated on passing current through it. This leads to errors that are generally difficult to calculate or compensate for. However, this can be avoided to some extent by allowing the current to flow in the circuit only when observations are being taken.
- **Defective or improper calibration** in instruments such as ammeter or voltmeter leads to errors in the measurement. In this case, there will be a constant difference between measured and true values. This arises due to manufacturing defect. The best option in such a situation is to calibrate the instrument against a standard equipment.
- **Faulty observation** can also arise due to **parallax**. To minimise error due to parallax, you should note the reading along the line, which is normal to both, the scale and the edge of the table on which scale is placed.

When an observer experiences relative movement of an object and its image, there exists a parallax between them.

I.4.2 Random Errors

You must have noticed that if the same measurement is repeated for the same quantity, you may get different readings with a scatter of values distributed about some mean value. These are called **random errors** and can arise due to accidental errors in the measurement process. The sources of random errors cannot always be identified. However we list a few possible sources.

- The **observational** random errors arise due to error of judgment of the observer while reading the smallest division in the scale (like the coincident vernier division with the main scale division). To minimise observational random errors, you should always take more readings and calculate their mean or draw the best fit graph as explained in Sec. II.2.1 in the next unit.

  If the values obtained in several measurements are x_1, x_2, x_3, …, x_N, the average value is determined by adding all the values and dividing their sum by the total number of observations:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N} \tag{I.1}$$

- The **environmental** random errors can arise due to unpredictable fluctuations in line voltage, sudden changes in temperature or mechanical vibrations, etc. There could also be a random spread of readings due to wear and tear or friction of mechanical part(s) of the system.

In the following SAQ you will classify some errors.

SAQ 5 – Classification of errors

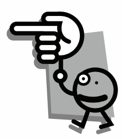
Classify the following according to the type of error involved by putting a tick in the appropriate column:

Sl. No.	Measurement	Type of Error	
		Systematic	Random
i)	A time interval measured using a stop watch that is running slow		
ii)	The length of a piece of steel rod is measured by several students in a laboratory		
iii)	A steel scale expands on a hot day to give a short reading of length		
iv)	The needle of a voltmeter is bent such that it does not rest on zero		
v)	The number of nuclear particles emitted per second by a sample of radioactive element		

When inexact values are used in a calculation, some error or uncertainty in the result is inevitable. The quality of a measurement and reliability of the result so obtained are determined by the magnitude of the estimated error. In scientific work, it is customary to quote a result along with associated error in measurement (with proper units) and upto the same order of magnitude.

We express the result of any measurement in a standard form along with error using the following rules:

1) **The error is stated up to *one significant digit* only.**
2) **The measurement is rounded off to the *same* order of accuracy as the error.**
3) **The result of measurement is written with the decimal point after *the first significant figure*.**
4) **The error is multiplied by *the same power as the measurement.***



Don't forget

If only rules (1) and (2) are followed, the form of result is correct but not standard. Rules (3) and (4) convert the result to standard form. For example, the standard form of result for measurement of length where metre scale is used should be written as (4.6 ± 0.1) cm.

Random errors can be quantified by statistical analysis and expressed as **absolute error** or **relative error**. Let us now learn how to estimate these.

I.5 ESTIMATING THE MAGNITUDE OF ERROR

Refer to Table I.3, where typical values of measurement of time period are given. Just by looking at the data, could you identify the "true" value of the time period? Probably not!

Table I.3: A set of typical values of measurement

Sl. No.	Data (x_i) (s)	Deviation $\Delta x_i = \lvert x_i - \bar{x} \rvert$ (s)
1.	2.69	0.01
2.	2.67	0.01
3.	2.68	0.00
4.	2.69	0.01
5.	2.68	0.00
6.	2.69	0.01
7.	2.66	0.02
8.	2.67	0.01
	$\bar{x}$ = 2.68	$\overline{\Delta x}$ = 0.009

The magnitude of the difference between the mean value of a physical quantity and its individual measured value (listed in the last column under 'Deviation' in Table I.3) is known as **absolute error** in the measurement. Let us denote it by Δx_i.

If *N* independent measurements of a quantity are labelled as $x_1, x_2, \ldots, x_N$, the average value is given by Eq. (I.1). In summation notation, we can write

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \text{(I.2)}$$

The symbol Σ (sigma) represents the sum of all measurements. As you can see from Table I.3, the average value of time period is 2.68 s. To calculate absolute error, we calculate the modulus of individual deviations $\Delta x_i = \lvert x_i - \bar{x} \rvert$ from the average value (as listed in the last column). Then, these deviations are added and their sum is divided by the total number of observations. Mathematically, we can write,

$$\overline{\Delta x} = \frac{\sum_{i=1}^{N} \lvert \Delta x_i \rvert}{N}. \qquad \text{(I.3)}$$

For the data given in Table I.3, the average value of absolute error is 0.009s. So we express the result of measurement as (2.68 ± 0.01) s. Note that absolute error has the same units as the quantity measured.

In error analysis, a useful measure of deviation is the ***variance***. That is, variance is a measure of the spread of a distribution of observations. For *N* observations, the variance in summation notation is given by

$$\sigma^2 = \frac{\sum_{i=1}^{N} (\Delta x_i)^2}{N} \qquad \text{(I.4)}$$

Once variance is known, its square root gives the **standard deviations**. It represents the range over which measurements vary. In other words, the standard deviation equals the magnitude of the uncertainty in the measurements.

You must be wondering as to why we consider standard deviation and not merely the average of deviations. This is because the individual deviations (which are also an indication of error involved in measurement) may be positive or negative. Since errors are additive in nature, it is more appropriate to take average of squares of the deviations and calculate standard deviation.

To give you a feel of the numbers, we would like you to answer the following SAQ.

SAQ 6 – **Standard deviation**

The measurement of the length of a table yields the following data:

x_1 = 135.0 cm, x_2 = 136.5 cm, x_3 = 134.0 cm, x_4 = 134.5 cm

Calculate the standard deviation and express the final result with possible error involved.

A better index of the accuracy of a measurement or equipment is **relative error**, also called percentage error. In fact, quite often we express our result by quoting the relative error rather than the absolute error. The relative error is the ratio of absolute error to the mean measured value of the quantity expressed in percent:

$$\delta x = \frac{\overline{\Delta x}}{\bar{x}} \times 100. \qquad \text{(I.5)}$$

Note that we have written the relative error as δx to distinguish it from the absolute error. You will note that the relative error covers all or most of the readings.

I.6 PROPAGATION OF ERRORS

You now know how to calculate error in the measurement of a directly observable physical quantity. But in most experiments, you would be required to measure two or more independent quantities to determine a physical quantity of interest. Therefore, the error in the final result depends on the errors in the measurement of individual parameters. In other words, the error in each measurement will "propagate" and get reflected in the final result. The actual analysis of propagation of errors is beyond the scope of this laboratory work. We shall, therefore, quote only some useful rules.

I.6.1 Error Propagation in Basic Operations

To understand how error propagates through basic mathematical calculations, we first consider addition and subtraction of two or more numbers.

Addition and subtraction

Suppose that two physical quantities A and B have measured values $(A \pm \overline{\Delta A})$ and $(B \pm \overline{\Delta B})$, respectively, where $\overline{\Delta A}$ and $\overline{\Delta B}$ are their absolute errors. Let us calculate the error $\overline{\Delta Z}$ in their sum $Z = A + B$.

We have by addition

$$Z \pm \overline{\Delta Z} = (A \pm \overline{\Delta A}) + (B \pm \overline{\Delta B}).$$

The maximum possible error in Z is therefore

$$\overline{\Delta Z} = \overline{\Delta A} + \overline{\Delta B}.$$

For the difference $Z = A - B$,

$$Z \pm \overline{\Delta Z} = (A \pm \overline{\Delta A}) - (B \pm \overline{\Delta B})$$

$$= A - B \pm \overline{\Delta A} \pm \overline{\Delta B}$$

or

$$\pm \overline{\Delta Z} = \pm \overline{\Delta A} \pm \overline{\Delta B} \qquad \text{(I.6)}$$

The maximum value of the error $\overline{\Delta Z}$ is sum of individual errors ($\overline{\Delta A} + \overline{\Delta B}$). Hence the rule for propagation of errors for a **sum or a difference** is: **The absolute error in the final result is the sum of the absolute errors in individual quantities.**

As such, Eq. (I.6) over-estimates the error. A more useful expression for $\overline{\Delta Z}$ based on statistical analysis is

$$\overline{\Delta Z} = \sqrt{\left(\overline{\Delta A}\right)^2 + \left(\overline{\Delta B}\right)^2} \qquad \text{(I.7)}$$

Let us now calculate a propagating error in the following example.

EXAMPLE I.1: PROPAGATION OF ERROS IN ADDITION

The measured values of two lengths L_1 and L_2 are (1.746 ± 0.001) m and (1.507 ± 0.001) m, respectively. Calculate the total error in the measurement of $L_1 + L_2$.

Solution

The error in measurement would be equal to the sum of errors in L_1 and L_2. Thus

$$\overline{\Delta L} = \overline{\Delta L_1} + \overline{\Delta L_2} = (0.001 + 0.001)\ \text{m} = 0.002\ \text{m}$$

and you can express the result as

$$L = (3.253 \pm 0.002)\ \text{m}$$

If you use statistical analysis, you will obtain

$$\overline{\Delta L} = \sqrt{\left(\overline{\Delta L_1}\right)^2 + \left(\overline{\Delta L_2}\right)^2}$$

$$= \sqrt{(0.001\text{m})^2 + (0.001\text{m})^2}$$

$$= 0.0014$$

$$= 0.001\ \text{m}$$

Note that we have kept only positive root because errors are cumulative.

Multiplication and division

If a quantity $E = A \times B$ and the results of measurement of A and B are ($A \pm \overline{\Delta A}$) and ($B \pm \overline{\Delta B}$), respectively, then we an write

$$E \pm \overline{\Delta E} = (A \pm \overline{\Delta A}) \times (B \pm \overline{\Delta B})$$

$$= AB \pm B\,\overline{\Delta A} \pm A\,\overline{\Delta B} \pm \overline{\Delta A}\,\overline{\Delta B}.$$

Dividing by $E = AB$ throughout, we obtain

$$1 \pm \frac{\overline{\Delta E}}{E} = 1 \pm \frac{\overline{\Delta A}}{A} + \frac{\overline{\Delta B}}{B} + \frac{\overline{\Delta A}\,\overline{\Delta B}}{AB}. \qquad (I.8)$$

Since $\overline{\Delta A}$ and $\overline{\Delta B}$ are small, the term $\frac{\overline{\Delta A}\,\overline{\Delta B}}{AB}$ can be neglected. Hence the maximum error in E is given by

$$\frac{\overline{\Delta E}}{E} = \frac{\overline{\Delta A}}{A} + \frac{\overline{\Delta B}}{B}. \qquad (I.9)$$

Let us now consider the propagation of error when the operation of 'division' is carried out. If we write $E = \frac{A}{B}$, the error $\overline{\Delta E}$ will also be given by Eq. (I.9).

If you take logarithm of $E = AB$ and differentiate it, you will get $\frac{\overline{\Delta E}}{E} = \frac{\overline{\Delta A}}{A} + \frac{\overline{\Delta B}}{B}$. This is generally known as the **logarithmic error**.

If you make statistical analysis, you will get the following result:

$$\frac{\overline{\Delta E}}{E} = \sqrt{\left(\frac{\overline{\Delta A}}{A}\right)^2 + \left(\frac{\overline{\Delta B}}{B}\right)^2}. \qquad (I.10)$$

You can now conclude that **when independent measurements are multiplied or divided, the fractional error in the result is the square root of the sum of the squares of fractional errors in individual quantities.** These results hold for absolute errors as well as relative errors.

Let us now see how error propagates in calculations involving operations of both multiplication and division.

EXAMPLE I.2 : ERROR PROPAGATION IN MULTIPLICATION AND DIVISION

A particular physical quantity, in an experiment, is computed from the relation $B = \frac{XY}{Z}$. Take the values of X, Y and Z measured in the laboratory as:

$X = 17 \pm 10\%$

$Y = 100 \pm 6$

and $Z = 15 \pm 3$

Let us see how error propagates in such a situation.

Solution:

By converting the uncertainties to percentage, you will find that

$$B = \frac{(17 \pm 10\%) \times (100 \pm 6\%)}{(15 \pm 20\%)}$$

$$= 113.33 \pm 36\%,$$

where we have added the uncertainties.

Proceeding further, you will note that 36% of 113.33 = 40.7988 so that

$$B = 113.33 \pm 40.80$$

This means that the uncertainty in the value of B is about 41. So, it makes no sense to retain the digits after the decimal in the value of B as well as the uncertainty. It is therefore more sensible to write:

$$B = 113 \pm 41.$$

I.6.2 Error Propagation in Angular Measurements

In your B.Sc. physics laboratory, you will get an opportunity to make very accurate and high precision measurements of angles. This is particularly true of experiments involving measurement of small physical quantities such as wavelength using a grating and a spectrometer. A spectrometer has a fixed circular protractor with a vernier moving over it. Usually the least count of a spectrometer is 1 min of an arc or $\frac{1}{16}^{\text{th}}$ of a degree. The calculations of error propagation are the same as in other measurements, as illustrated below.

In a diffraction grating experiment, the wavelength $\lambda = N \sin\theta$, where N is the number of rulings in the grating and θ is the angle of diffraction for that λ. Then

$$\frac{\overline{\Delta\lambda}}{\lambda} = \frac{N\cos\theta}{N\sin\theta} = \cot\theta\,.$$

I.6.3 Error Propagation due to Exponent of a Measured Quantity

Suppose we have to calculate the area of a square piece of land of side A. It is given by $s = A \times A = A^2$. From Eq. (I.9), it readily follows that

$$\frac{\overline{\Delta s}}{s} = \frac{\overline{\Delta A}}{A} + \frac{\overline{\Delta A}}{A}$$

$$= 2\frac{\overline{\Delta A}}{A} \qquad \text{(I.11)}$$

That is, the error in A^2 is twice the error in A. You will obtain the same result if you take the logarithm of both the sides:

$$\log s = 2 \log A$$

On differentiating and changing the differentials to 'deltas', we get Eq. (I.11).

For a wire, the diameter d is measured as (1.02 ± 0.01) mm. Therefore, the error in the area of cross section $\left(= \frac{\pi d^2}{4}\right)$ will be twice the error in d, i.e. nearly 2%.

In general, if a quantity appears in an expression with a power n (> 1), its error contribution increases n-fold. This means that you should measure quantities appearing with power 2 or more with a higher degree of accuracy. Moreover, take a large number of readings. In case its magnitude is small, you should take readings at different points/ perpendicular directions.

Now study the following example.

EXAMPLE I.3: ERROR PROPAGATION DUE TO EXPONENTS

The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. L is about 100 cm and is known to 1 mm accuracy. The period of oscillation is about 2 s. The time of 100 oscillations is measured with a wrist watch having least count of 1 s. Calculate the percentage error in the value of g.

Solution

You can rewrite the expression for T as

$$g = \frac{4\pi^2 L}{T^2}$$

Therefore, the percentage error in g can be calculated using the relation

$$100\,\overline{\frac{\Delta g}{g}} = 100\,\overline{\frac{\Delta L}{L}} + 2 \times 100\,\overline{\frac{\Delta T}{T}}$$

The percentage error in $L = 100\,\overline{\frac{\Delta L}{L}} = 100 \times \frac{0.1\text{ cm}}{100\text{ cm}} = 0.1\%$

and

the percentage error in $T = 100\,\frac{\overline{\Delta T}}{T \times n} = 100 \times \frac{1\text{s}}{2 \times 100\text{ s}} = 0.5\%$

Hence

$$100\,\overline{\frac{\Delta g}{g}} = 0.1 + 2 \times 0.5 = 1.1\%$$

Note that we have multiplied T by n while calculating the percentage error in T. Do you know why? This is because the actually measured quantity is time for 100 oscillations ($n \times T$) rather than T. If you take time for one oscillation, the percentage error in T will be 50%. It means that **taking more oscillations per observation helps us to reduce error in a measurement.**

Let us now sum up what you have learnt in this unit.

I.7 SUMMARY

- Precision of any measurement depends on the least count of the measuring instrument.
- The result of every measurement is expressed in numbers such that only the last digit contains error.
- In scientific notation, a measurement is expressed as a decimal number between one and ten multiplied by appropriate power of ten.
- **Relative error** is the ratio of probable error to total measurement. Accuracy is related to relative error.
- **Systematic errors** can arise due to zero error, backlash error, end correction, defective calibration or faulty observation procedures. Such errors are identifiable. So these can be eliminated or accounted for.
- **Random errors** can arise due to error of judgement and environmental factors during the performance of measurements. Such error results in a scatter of values and to minimise these, we take a large number of observations.
- While adding (or subtracting) approximate numbers, round off the sum (or difference) to the same unit of measure as the least precise measurement.
- The magnitude of errors can be computed statistically. It is usually expressed as a mean of deviations of observed values from the final value or through standard deviation.
- Errors are cumulative and propagate in an experiment depending on the number of measurements and measuring devices.

I.8 TERMINAL QUESTIONS

1. A physical quantity x is related to three other physical quantities a, b and c through the relation

   $$x = ab^2c^{-3}$$

   If the errors in a, b and c respectively are 1%, 3% and 2%, calculate the percentage error in x.
2. In the measurement of viscosity of a liquid, we determine the rate of flow of the liquid (volume flowing per second, V) through a capillary tube of radius

a and length *L* under constant pressure difference *p*. The expression for viscosity is given by

$$\eta = \frac{\pi p a^4}{8LV}.$$

If the percentage errors in *p*, *a*, *V* and *L* respectively are 1%, 1%, 2% and 1%, calculate the percentage error in η.

I.9 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. a) 19.87 cm
   b) 3.21 s
   c) Equally precise

2. a) The relative errors are:

   $$\frac{0.05}{40} = \frac{5}{4000} = \frac{1}{800}$$

   and

   $$\frac{0.05}{8} = \frac{5}{800} = \frac{1}{160}$$

   Therefore, the measurement 40.0 cm is more accurate. However, both measurements are equally precise.

   b) The measurement 0.85 m is more accurate but as precise as 0.05 m.

3. 3×10^{-23} g

4.

Sl. No.	Measurement (m)	No. of significant digits	Unit of measurement (m)	Probable Error (m)	Relative error
1.	0.2	1	0.1	0.05	$\frac{0.05}{0.2} = 0.25$
2.	0.20	2	0.01	0.005	$\frac{0.005}{0.20} = 0.025$
3.	0.2000	4	0.0001	0.00005	$\frac{0.00005}{0.2000} = 0.00025$
4.	25	2	1	0.5	$\frac{0.5}{25} = 0.02$
5.	250	3	1	0.5	$\frac{0.5}{250} = 0.002$
6.	25000	5	1	0.5	$\frac{0.5}{25000} = 0.00002$
7.	102	3	1	0.5	$\frac{0.5}{102} = 0.0049$
8.	1002	4	1	0.5	$\frac{0.5}{1002} = 0.000499$

   a) They are significant.

   b) They are also significant. As a rule, only those zeros are significant which come from a measurement. Since the unit of measurement is

1 m in both these cases, the zeros trailing the numbers are arising out of the measurement and hence are significant.

c) Significant.

5. i) systematic, ii) random, iii) random

iv) systematic v) random

6.

Sl. No.	Length (x_i) (cm)	$\Delta x_i = x_i - \bar{x}$ (cm)	$(\Delta x_i)^2$ (cm^2)
1.	135.0	0	0
2.	136.5	+1.5	2.25
3.	134.0	−1.0	1.0
4.	134.5	−0.5	0.25

$$\bar{x} = \frac{\sum x_i}{N} = \frac{540}{4} = 135.0 \qquad \sum(\Delta x_i)^2 = 3.5$$

$$\sigma = \sqrt{\frac{\Sigma(\Delta x_i)^2}{N}} = \sqrt{\frac{3.5}{4}} = \sqrt{0.9} \approx 1.0\,\text{cm}.$$

The final result can be expressed as length = (135 ± 1 cm)

Terminal Questions

1. To calculate the percent error, we note that

$a = (a_0 \pm 1\%)$

$b = (b_0 \pm 2 \times 3\%)$ and

$c = (c_0 \pm 3 \times 2\%)$

So the total percentage error in x is1+6+6 = 13%.

2. $$\frac{\Delta\eta}{\eta} = \frac{\Delta p}{p} + 4x\frac{\Delta a}{a} + \frac{\Delta V}{V} + \frac{\Delta l}{l}$$

$\therefore$ Percentage error in η is = 1+4+2+1 = 8%.

UNIT II

GRAPHING

Structure

II.1 Introduction
Expected Learning Outcomes
II.2 Plotting a Graph
Linear Plots
Non-linear Plots
II.3 Error Estimation on Graphical Plots
II.4 Summary
II.5 Terminal Questions

II.1 INTRODUCTION

In Unit I you learnt that the result of any measurement is expressed in the form of numbers. When we perform any experiment, generally we try to establish a relationship between two or more physical quantities to arrive at the results.

Many times, it is not possible for us to visualise the functional relationship between two quantities by just looking at the experimental data. But if we plot a graph, it becomes very easy, quick and convenient to predict the nature of relationship. Once such relationship is known, we can even predict the value of a parameter for intermediate values of the quantities, where we may not have taken actual observation. In fact, graphs can also be used to minimise errors or locate inaccuracy in observations.

In many of the experiments you will be performing in this course, you will need to plot the graphs of various quantities. There are certain good practices of plotting a graph, which makes it readable and understandable to everybody. In this unit you will become familiar with methods of plotting the graphs and writing correct legends. Hence we will advice you to go through this unit carefully before starting the experiments in the laboratory.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ establish functional relationship between various physical quantities;
- ❖ make a choice of appropriate scale on the graph;
- ❖ depict the observations on the graph with error bar;
- ❖ use the criterion of best fit in a straight line plot;
- ❖ interpret a graph and determine the values of physical quantities of interest; and
- ❖ carry out error estimation in slope and intercept on a plot.

II.2 PLOTTING A GRAPH

In this section, we describe both linear and non-linear plots.

II.2.1 Linear Plots

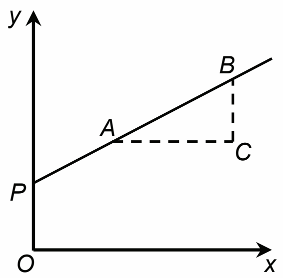


Fig. II.1: A straight line graph.

A straight-line graph is the easiest to draw (Fig. II.1). The equation for a straight line is $y = mx + c$, where m is the slope (gradient) and c is the intercept on the y-axis. In Fig. II.1, the slope of the straight line is given by

$$m = \frac{BC}{AC}$$

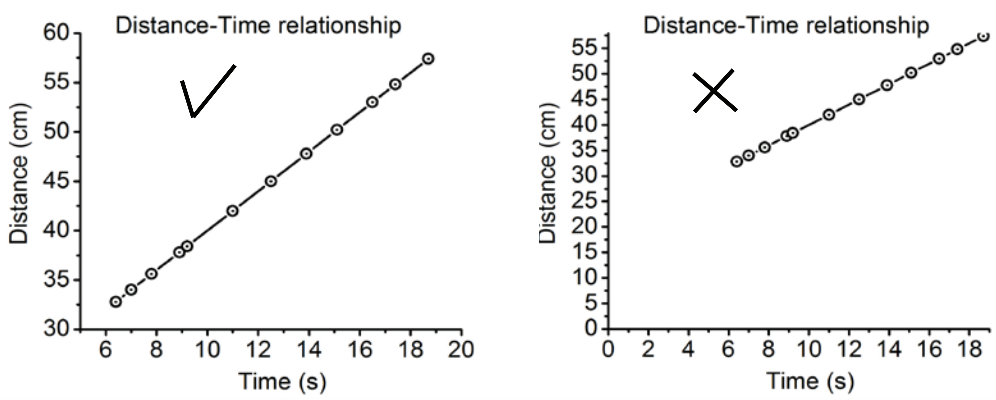
and OP is its intercept c.

You should use a graph-paper to draw the graphs. Generally the graph-paper has a grid of 1 cm × 1 cm squares printed on it. Each square is further divided into 1 mm × 1 mm sub-parts. Such graph paper is referred to as linear graph paper.

When drawing graphs, you must observe the following points:

i) Identify the independent and dependent variables. It is a customary to plot the independent variable along the x-axis and the dependent variable along the y-axis.

ii) You should choose the scales so that the points are suitably spread out on the entire graph paper as shown in Fig. II.2a rather than being cramped into a small portion as done in Fig. II.2b. For this, first of all note the minimum and maximum values of the data to be plotted. Then round off these numbers to slightly less than the minimum and slightly more than the maximum. The resulting difference should be divided by the number of divisions on the graph paper. For example, if you are to plot data between 6.4 s and 18.7 s on x-axis and corresponding y-axis readings range between 32.8 cm and 57.4 cm then, it would be convenient to allow the x-scale to run from 5 to 20 s rather than 0 to19 s and y-scale between 30 and 60 cm instead of 0 and 58 cm.



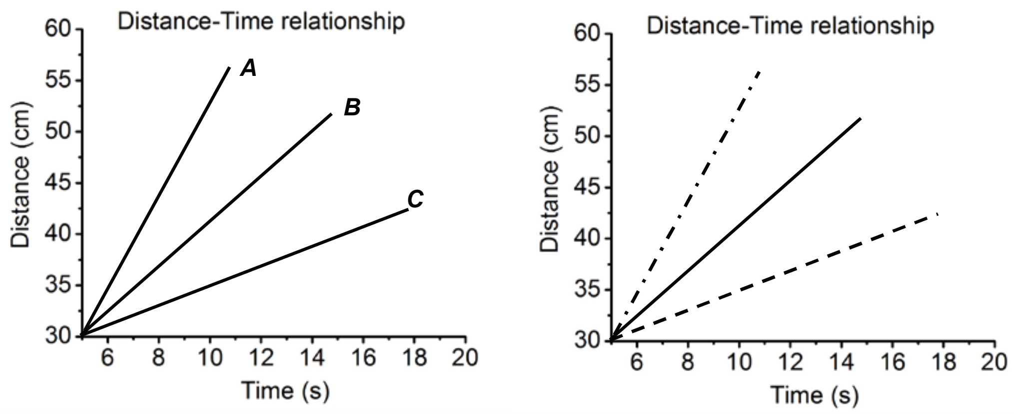
(a) (b)

Fig. II.2: Choice of scale a) proper; and b) improper.

iii) Draw **axes** clearly and write the **name** of the physical quantity to be plotted, its symbol, unit and the scale used along each axis.

iv) Use a plotting symbol such as a dot and encircle it to show the measured position of points (See Fig. II.2a). In no case, the size of this circle should exceed the size of the smallest square on the graph paper.

v) You should give the graph a suitable **caption**.



(a) (b)

Fig. II.3: Drawing graphs with more than one curve.

vi) If there is more than one curve on the graph, label different curves (Fig. II.3a). Alternatively, you can use different notations (dash dot, solid, dash) to show different curves (Fig. II.3b).

vii) The curve drawn should be the simplest mean curve that fits the data. In the graph shown in Fig. II.4, it is easy to see that the data points lie on a straight line. This is referred to as best-fit curve. Note that the line may not necessarily pass through each observed point. However, it should pass through the region of uncertainty for each point. This region of uncertainty is depicted as an **error bars** (small vertical line representing the uncertainty in the measurement) around each point in the figure.

> Though drawing a best fit curve for non-linear data may involve extensive statistical treatment, the method explained here gives fairly good fit of the linear data being plotted.

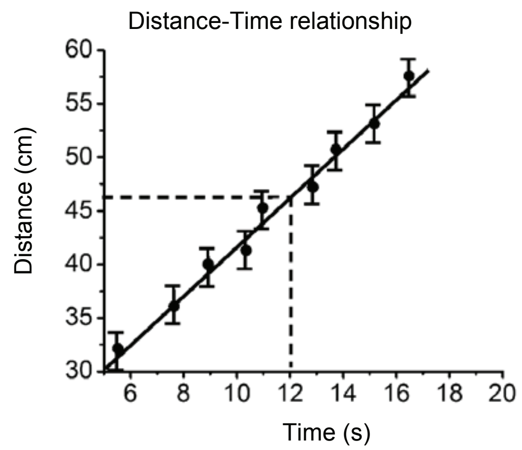


Fig. II.4: A best fit curve.

You can use this plotted curve to determine the value of a parameter, where the reading is not taken by you. For example, in Fig. II.4, there is no observation corresponding to time 12 s. However, you can conclude by following the plotted curve that the distance of interest is 46 cm.

You will use these graphing techniques in some of the experiments of this course such as the experiments on determination of Young's Modulus by bending of beam apparatus and spring constant by spring-mass system.

II.2.2 Non-linear Plots

In Fig. II.4, we plotted distance along *y*-axis and time along *x*-axis on a linear graph paper, since these quantities are linearly related. In some experiments, however, we may get data where the relationship between the measured variables is not linear and we have to plot a graph where the variables of interest are related through a power-law. For example, in a simple pendulum, the time and length are related by equation $T = k\ell^{1/2}$. In such cases, to draw a graph between the time period and the length of the pendulum, you will have to calculate square root for each value. This introduces another step in the procedure, and is obviously cumbersome.

Many a times, the variables in a relation may vary over different powers. For example, the voltage and current relationship for a forward biased semiconductor diode is given by the equation:

$$I_D = I_S\left[\exp\frac{qV_D}{kT} - 1\right] \qquad \text{(II.1)}$$

The direct plot of I_D Vs V_D will result into an exponential curve, however, if we take the logarithm of current values, then, its plot with V_D will result into linear curve, as shown in Fig. II.5. Such plot is called a semi-log plot.

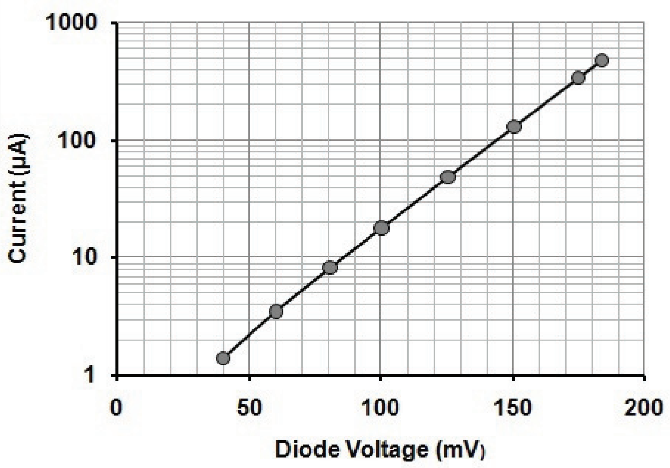


Fig. II.5: Semi-log plot of diode current vs. voltage.

Another example is found in Astronomy. According to Kepler's law, the period of a planet T (time for one revolution around the sun) is related to the semi-major axis of it's orbit (R) by the relation

$$R^3 = kT^2, \qquad \text{(II.2)}$$

where k is constant.

If you consider the experimental data which shows how T depends on R, you will observe that the latter varies by three orders of magnitude and T varies by two orders of magnitude. In other words, the experimental data follows

Eq. (II.2). For a moment suppose you do not know the exact relationship between the variables T and R. Then you can write

$$R = kT^{n}, \qquad (II.3)$$

where n is constant. In such cases, you can obtain the value of n by taking logarithm of (Eq. II.3):

$$\log R = \log k + n \log T \qquad (II.4)$$

Now you can plot log R versus log T on a linear graph paper. The slope of straight line obtained will give the value of exponent n. But again, as mentioned above, taking logarithm of each experimental data is rather tedious. For convenience, we use a log-log (Fig. II.6) graph paper in such cases, so that the resultant curve is a straight line and we can easily calculate its slope and intercepts.

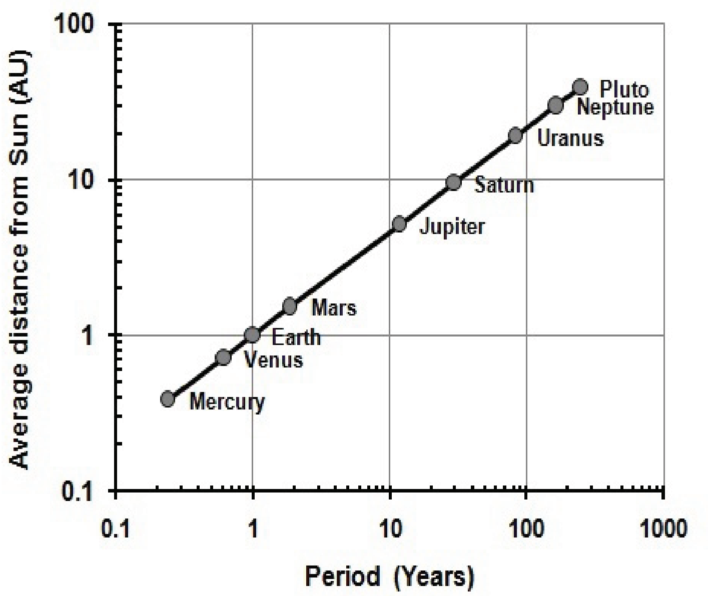


Fig. II.6: Log-log graph of planet's average distance from the Sun versus its period of revolution around the Sun.

In your first year lab, you may not be required to use these and we therefore end our discussion here.

II.3 ERROR ESTIMATION ON GRAPHICAL PLOTS

In Fig. II.7 we have plotted the velocity, V, versus time period, T, which represent a linear relationship between them. The result is plotted along with the error bar around each data point.

To determine the error in the value of the slope of the straight line drawn on a graph paper (linear, semi-log or log-log), you should draw two lines representing the greatest and the least possible slopes which reasonably fit the data.

For the graph in Fig. II.7, the error in the slope is defined as

$$\text{error in slope} = \frac{\text{maximum slope} - \text{minimum slope}}{2}$$

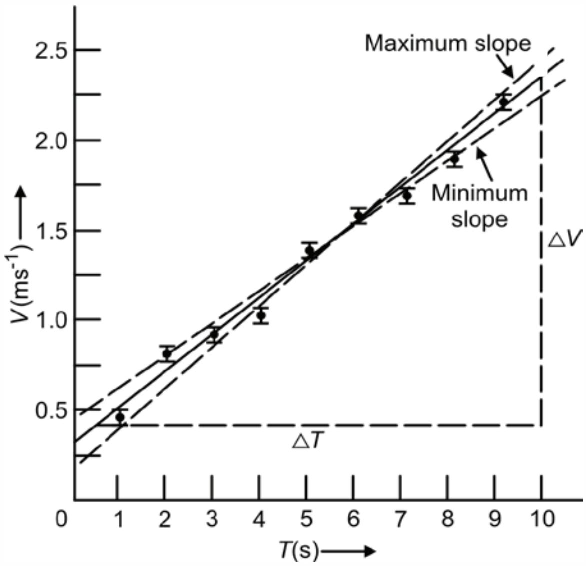


Fig. II.7: Graph of velocity and time period with error bars.

Similarly,

$$\text{error in intercept} = \frac{(\text{intercept of minimum slope line} - \text{intercept of maximum slope line})}{2}$$

Let us now sum up what you have learnt in this unit.

II.4 SUMMARY

- Graphical representation of observations eases data interpretation;
- A functional relationship, especially, a linear relationship, between the two experimental parameters can be established by visual inspection;
- Log-log or semi-log plots can be used to depict non-linear relationships;
- Unknown parameter value within the experimental range of observation can be deduced by plotting a graph; and
- Error in the slope and intercept can be obtained from a graphical representation.

II.5 TERMINAL QUESTIONS

1. Draw the graph of the equation: $y = 2x + 4$. Draw the best fit straight line and find its slope.
2. Plot the graph of y versus x for the following set of observation. Draw the best fit straight line and obtain its slope. Write the equation of the line you have drawn.

x	0	1	2	3	4	5	6	7	8
y	4.1	6.1	7.9	10.0	11.9	13.9	16.0	18.1	20.0

3. Plot the graph of *y* versus *x* for the following data and draw the best fit straight line. Determine the slope and the equation of the straight line.

x	10	11	12	13	14	15	16	17	18
y	24	26.1	27.8	30	32.2	34	35.8	38.2	40

EXPERIMENT 1

MEASUREMENT OF LENGTH

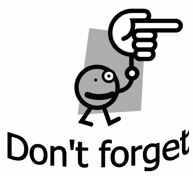
Structure

1.1 Introduction
  Expected Skills
1.2 Measurement of the Thickness of a Wooden Block using Vernier Callipers
  Working with Vernier Callipers
  Measurements using Vernier Callipers
1.3 Measurement of the Thickness of a Paper Sheet using a Screw Gauge
  Working with a Screw Gauge
  Measurements using Screw Gauge
1.4 Measurement of the Internal Diameter of a Capillary using Travelling Microscope
  Working with Travelling Microscope
  Measurements using Travelling Microscope

1.1 INTRODUCTION

You must have performed various experiments in a physics laboratory during your school days. You know that it is necessary to make different measurements while doing these experiments. The simplest measurement is that of length. When we wish to know the dimensions of a room or a piece of a land, we use a measuring tape; and we use a metre scale when we buy some cloth. You would be familiar with such measurements and may have used a measuring tape or a metre scale.

In the laboratory, however, you will need to measure small lengths, say, the diameter of a bob or a metal wire or the diameter of a capillary. These require accuracy better than that obtained with a metre scale – of the order of 0.01 cm or even less. For measuring small lengths, we use devices like vernier callipers and screw gauge, depending on the accuracy required. In some special cases, we also use travelling microscope. In this experiment, you will learn how to use all these instruments for measuring length. You should remember for all measuring instruments (used for measuring length or any other quantity) that,



- **no measurement can be more accurate than the precision of the measuring instrument; and**
- **there is a limitation on the accuracy with which data can be taken.**

This means that **a measurement can never be exact** and there will always be deviations from the true value. That is, some uncertainty (error) is always present in every measurement. So before you make measurements with any instrument, you must have a clear idea of the concept of errors we have discussed in Unit I of this course. (You will learn that we always quote the result along with the error.)

In the first part of this experiment you will learn how to measure the thickness of a wooden block using **vernier callipers** (Sec. 1.2). Then you will learn how to use a **screw gauge** to measure the thickness of a sheet of paper or the diameter of a wire (Sec. 1.3).

Sometimes in the physics laboratory you have to measure very small distances accurately, for example, the width of an interference fringe or the height to which water rises in a capillary tube or a small displacement of a needle due to bending of beam. For this we use a type of the compound microscope called the **travelling microscope**. In the third and final part of this experiment, you will learn how to focus a travelling microscope and make measurements using it (Sec. 1.4). You will find out that while measuring the lengths with the help of a travelling microscope, it does not physically touch the object under study, unlike the vernier callipers or the screw gauge. Hence, here you will be performing length measurements in the non-contact mode.

Expected Skills

The purpose of this experiment is to train you in handling the instruments used to measure small lengths. After doing this experiment, you should be able to:

- ❖ obtain the least count and estimate the zero error of vernier callipers and use it to determine the thickness of an object;
- ❖ obtain the least count and estimate the zero error of a screw gauge and use it to determine the thickness of a paper sheet or diameter of a wire; and
- ❖ focus a travelling microscope and use it to make small length measurements in the non-contact mode.

The apparatus required for this experiment is listed below.

Apparatus required
Vernier callipers, screw gauge, travelling microscope, a wooden block, metallic wire/needle or a sheet of paper, a piece of glass capillary, a spirit level and a stand with cork clamp.

In the first part of the experiment, we describe the vernier callipers, how to find its least count and zero error, if any. You will also learn the procedure for measuring the thickness of a wooden block using it.

1.2 MEASUREMENT OF THE THICKNESS OF A WOODEN BLOCK USING VERNIER CALLIPERS

When you wish to measure lengths in the range of 5 mm to 10 cm, with a precision better than 1 mm, then you should use vernier callipers. You will be using the vernier callipers in some experiments in the laboratory; e.g. to measure the radius of the metal bob used in simple pendulum, the inner / outer diameters of the cylinders used as weights in torsional pendulum. In the present experiment, you will measure the width of a wooden block using the vernier callipers. Before you perform the actual experiment, it is important for you to know about the construction and working of vernier callipers.

1.2.1 Working with Vernier Callipers

The vernier callipers is a steel apparatus which has two jaws *A* and *B* as shown in Fig.1.1. Jaw *A* is fixed to a scale (main scale) of about 15 cm length with millimetre markings on it. Jaw *B* is attached to a small movable scale called the vernier scale, V. The object, whose length (a wooden block in our case) is to be measured, is held between these two jaws. Measurement of length is done by following the procedure given later in this section.

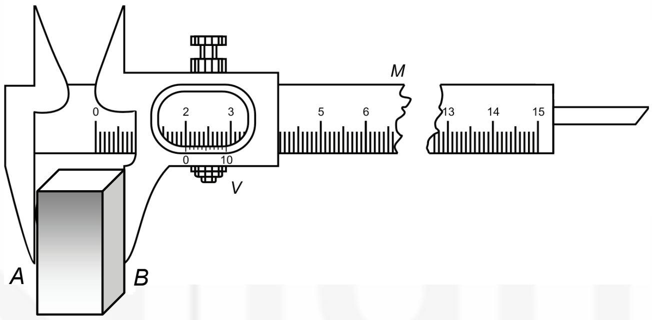


Fig.1.1: Measurement of the thickness of a wooden block with vernier callipers

While using any measuring instrument, it is very important to know its **least count** and any systematic error caused due to instrument setting. In case of length measuring instruments, there may be a mismatch between the zero markings of the main scale and the moving scale. This is called the **zero error** of the instrument. It is a systematic error, which can be corrected by adding (or subtracting) it from the reading you are taking. Now we will discuss, in brief, the method of obtaining the least count and zero error of vernier callipers.

a) Least count

In simple vernier callipers, the vernier scale has 10 divisions which are equal to 9 divisions or 9 mm of the main scale (Fig. 1.2). Thus, the value of each vernier division is 0.9 mm and it is 0.1 mm shorter than one main scale division. **When the jaws are closed so as to touch each other, the zero of**

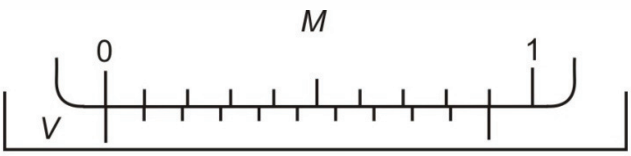


Fig.1.2: Least count of a vernier callipers.

the main scale should coincide with the zero of the vernier scale. Since a vernier division is shorter than a main scale division by 0.1 mm, the first vernier division will lie 0.1 mm left to the first main scale division. Now if you move the jaw *B* slowly to the right, such that this 0.1 mm difference disappears, the jaw should have moved by 0.1 mm. This would also be the width of the gap opened between the two jaws. So, the smallest length that can be measured by the vernier callipers is 0.1 mm or 0.01 cm. This is called the **least count** or the **vernier constant** of the instrument (vernier callipers).

> *The least count of vernier callipers is equal to the difference between the lengths of one main scale division and one vernier scale division.*

You will now learn to calculate the least count of vernier callipers. This method can be used for any instrument carrying a pair of MS (main scale) and VS (vernier scale).

Calculating the least count of vernier callipers

The least count (LC) of a vernier callipers is defined as

LC = Value of one Main Scale Division (MSD) − Value of one Vernier Scale Division (VSD) in terms of MSD

Study Fig. 1.2. Note that the value of 10 VSD is equal to the value of 9 MSD. Therefore, we can say that in this case 1 VSD = (9/10) MSD. Hence, we can write the expression for least count as

$$\text{LC} \equiv 1\text{MSD} - 1\text{VSD} = 1\text{MSD} - (9/10)\ \text{MSD} = (1/10)\ \text{MSD}$$

For the vernier callipers shown in Fig.1.2, the value of 1MSD = 1 mm. Therefore,

$$\text{LC} = \left(\frac{1}{10}\right)\text{mm} = 0.1\,\text{mm} = 0.01\,\text{cm} \qquad (1.1\text{a})$$

In your physics laboratory, you may come across some instruments in which the vernier scale has more than 10 divisions and the value of 1 MSD is less than 1 mm. One such familiar example is that of **travelling microscope**, which you will handle in the later part of this experiment. On the travelling microscope scale, 49 MSD correspond to 50 VSD and 1 MSD = 0.5 mm. Therefore, the least count of the travelling microscope is

$$1\ \text{MSD} - (49/50)\ \text{MSD} = (1/50)\ \text{MSD} = (0.5/50)\ \text{mm} = 0.01\ \text{mm} = 0.001\ \text{cm} \qquad (1.1\text{b})$$

Note that the denominator in the brackets of Equations (1.1a and b) is equal to the total number of divisions on the VS. Therefore, in general, we can write

$$\text{LC} = \frac{1}{n} \times \text{value of 1 MSD} \qquad (1.2)$$

where n is the total number of divisions on the VS. Since n = 50 and 1 MSD = 0.5 mm for a travelling microscope, we can write its least count (LC) as

$$\text{LC} = \frac{1}{50} \times 0.5\,\text{mm} = 0.01\text{mm} = 0.001\text{cm}$$

Other instruments to which a vernier scale is fitted include spherometer, a Fortin's barometer and a spectrometer. You will work with these instruments while doing the experiments in your laboratory course.

b) Zero error

When the jaws of the vernier callipers are in contact, the zero mark of the MS should coincide with the zero mark of VS (see Fig.1.3a). In some vernier callipers this may not happen. In such a case, the vernier callipers is said to have a **zero error**. The zero error must be determined and accounted for while taking measurements with the vernier callipers.

Suppose that when the jaws *A* and *B* touch each other, the zero of vernier scale lies **to the right** of the main scale zero (Fig.1.3b). This error is called **positive zero error**. To obtain the correct measurement we have to apply **zero correction**. How do we do it? If the m^{th} division of the vernier scale coincides with a main scale division, the instrument is showing a reading equal to *m* times the least count. The magnitude of the positive zero error in this case is *m* x LC.

If the instrument (vernier callipers) has positive error, the zero error has to be subtracted from any reading taken by the callipers.

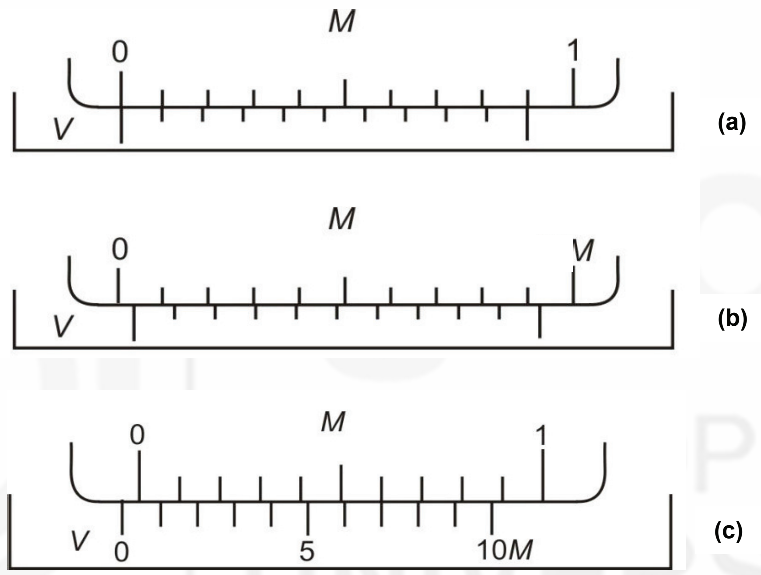


Fig.1.3: a) No zero error as zero marks of MS and VS coincide; b) positive zero error; c) negative zero error.

In case the zero of the vernier scale falls on the left of the zero of the main scale, the vernier callipers is said to have **negative zero error** (Fig.1.3c). Then

$$\text{Magnitude of the negative zero error} = (n - m) \times \text{LC}$$

where *n* is the total number of divisions on the vernier scale and the m^{th} vernier scale line coincides with a main scale line. For example, in Fig. 1.3c, $m = 7$, and hence the magnitude of negative error is $(10 - 7) \times \text{LC} = 3 \times \text{LC}$.

In case of negative zero error, we apply correction by adding a value equivalent to the error, to the observed reading of the instrument.

We assign positive zero error with + sign and negative zero error with – sign. So a general rule is that we always **subtract the zero error with proper sign from the observed reading.**

In your school curriculum, you may have learnt about the vernier callipers. In order to refresh your knowledge, perform the following steps:

- Identify the main scale (MS) and the vernier scale (VS) on the vernier callipers and write the number of divisions on VS:

  No. of divisions on the vernier scale =…………………………………..

- Note how many of MS divisions equal all VS divisions:

  …. No. of divisions in the main scale = …. No. of division on vernier scale

Now calculate the least count of your vernier callipers as follows:

- Calculate 1 VSD = $\frac{\text{No. of divisions on main scale}}{\text{No. of divisions on vernier scale}} = \ldots\ldots\ldots \text{MSD}$

- Note the value of 1 division on main scale = 1 MSD = …….. mm.

- Calculate the least count of vernier callipers using the formula:

  Least Count = Value of (1 MSD − 1 VSD) = ………… mm.

After finding out the least count of the vernier callipers, you will now use it to measure the length of some small objects. For this part of the experiment, you will need vernier callipers and a solid object like small piece of wood.

1.2.2 Measurements using Vernier Callipers

You should now follow the steps given below to measure the thickness of the wooden block:

1. Bring the jaws of the vernier callipers in contact and note whether or not the zeroes of the VS and MS coincide. In case they do not coincide, then it possesses a **zero error**. **Do not push the jaws together forcefully to make them coincide. Doing so may damage the callipers**. Find out the zero error as described above and record it in Observation Table 1.1 with appropriate sign (+ for positive and – for negative error). Remember that zero error, whether positive or negative, is **taken with its sign** and always **subtracted** from each measured value.
2. Record the least count in Observation Table 1.1.
3. Hold the block between the jaws, as shown in Fig.1.1.
4. Slide the vernier scale so that the jaw of the vernier scale touches the other face of the block.
5. The position of the zero mark of the vernier scale, as read on the main scale, gives a rough estimate of the thickness of the block. If the zero mark of VS corresponds exactly to any particular marking on the MS, then that reading of MS is the exact reading of the length. However, if the zero mark on the vernier scale lies in-between the two markings, say, between 3.3 cm and 3.4 cm, as shown in Fig.1.4, then the thickness of the block is more than 3.3 cm (called the main scale reading), but less than 3.4 cm. You can find out how much more it is than 3.3 cm by noting the division on the VS that coincides with a MS division. If the fourth division on the VS coincides with an MS division, the thickness of the block would be

3.3 cm + 4 × 0.01 cm = 3.34 cm. (If the zero mark of VS corresponds exactly to any particular marking on the MS, read the main scale. This reading gives the thickness of the block.)

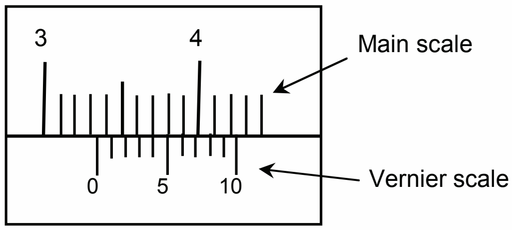


Fig.1.4: Reading a main scale and vernier scale.

In general, the distance between the two jaws of the vernier callipers is given by:

MS reading + (vernier reading × least count)

Record your reading in Observation Table 1.1. The graduations on the vernier scale are very fine and close together. Therefore, you may find it convenient to use a magnifying glass to take readings.

6. Repeat the steps 3 to 5 at least four times at different points on the same faces of the block. You must have realized that this exercise is to minimise random errors.
7. Subtract the zero error (with proper sign), if any, from each measured value to obtain correct value and note it in the Observation Table 1.1.
8. Calculate the mean of corrected values. This will give you the thickness of the given block.
9. Calculate the percentage error using the procedure explained in Unit-I and quote your result.

Note that vernier scale reading is just a number, while MS reading is in the units of length.

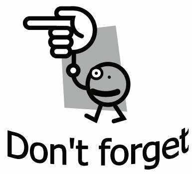
Observation Table 1.1: Measurement of thickness of a wooden block

Least count of the vernier Callipers = ………. cm

Zero error of the vernier Callipers = ……….. cm (with + or – sign)

Sl. No.	Main Scale (MS) reading (cm)	Vernier Scale (VS) reading	Thickness (cm)	
			Measured reading (=MS+LC×VS)	Corrected reading (Measured value – Zero error)
1.				
2.				
3.				
4.				

Result: The thickness of the given block is ………..**cm** ±………..**cm**



For proper handling of a vernier callipers, you should

- **not apply excessive pressure on the jaws or over-stress them while noting zero error or taking readings; and**
- **store them in the boxes provided by the manufacturer.**

On completing this part of the experiment, you will discover that vernier callipers can be used to measure lengths in the range 0-15 cm with an accuracy of 0.01 cm. When we need accuracy better than that obtained with vernier callipers, we use a screw gauge. In the next part of this experiment, you will work with a screw gauge.

1.3 MEASUREMENT OF THE THICKNESS OF A PAPER SHEET USING SCREW GAUGE

When you want to measure extremely small lengths like diameter of a wire or thickness of a paper, you need an instrument with better precision than that of the vernier callipers. Later in this laboratory course, you will use a screw gauge to measure the radius of thin wire and thickness of the beam in some experiments. In this part of the experiment, you will learn how to measure the thickness of a paper sheet (or diameter of a wire). But, before proceeding further, you should know the parts of the screw gauge.

1.3.1 Working with a Screw Gauge

Fig.1.5 shows a screw gauge, in which a screw moves in accurately cut grooves. Screw gauge consists of a spindle (screw), a U-shaped frame, a hollow shaft (sleeve) or barrel on which a linear scale is engraved with millimetre divisions. It acts as the main scale in the screw gauge. A cylindrical collar called a thimble, is attached to the spindle. A circular scale with 100 (or sometimes 50) divisions is engraved on this collar. When the thimble is rotated, it gives linear displacement to the spindle. On one side of the frame a flat stud called anvil is fitted. The object whose length is to be measured is held between the anvil end *A* and the spindle end *B*. You can tighten the hold properly by rotating the ratchet attached to the screw.

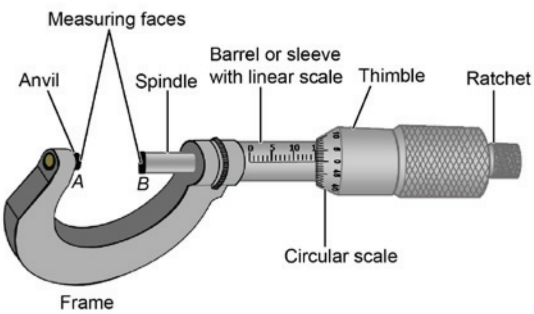


Fig.1.5: A screw gauge.

With a constant use, some wear and tear occurs in the movement of the screw on the grooves. As a result, it is also possible that there may not be forward linear motion of the screw until a certain rotation is given to the circular head. This lagging behind of the linear motion with respect to the circular motion is called **backlash error**. To avoid this, **you should always rotate the screw gauge in the same direction.**

Like any other measuring instrument, a screw gauge also has a least count and may sometimes possess positive or negative zero error. The three important parameters we should know about a screw gauge are: **pitch, least count** and **zero error**. We discuss these briefly before you start using this instrument.

a) Pitch

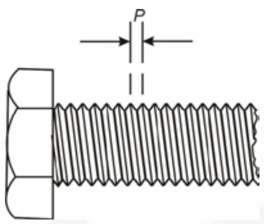


Fig. 1.6: Pitch of the screw gauge.

The screw of the spindle is the most important part of a screw gauge. It has very accurate threads cut on it which, on rotation, move the screw forward or backward. **The distance moved by the spindle in one complete revolution of the screw is called the pitch, *P*, of the screw gauge** (Fig.1.6). If we rotate the thimble clock-wise, the spindle will move towards the anvil end *A*. When the two touch each other, the zero mark on the circular scale should coincide with the zero mark of the main scale. Now, if we rotate the thimble by one complete anticlockwise rotation, the zero mark on the circular scale will once again coincide with the main scale mark, and the spindle end, *B* will be separated from the anvil end, *A*. You can read this separation on the main scale. It is equal to the **pitch** of the screw.

b) Least Count

Suppose the pitch of the screw is 0.5 mm and there are 50 divisions on the circular scale. Now, if the thimble is rotated only through one division on the circular scale, the distance moved by the spindle is:

$$(1/50) \times \text{pitch} = (1/50) \times (0.5\text{mm}) = 0.01 \text{ mm or } 0.001 \text{ cm}$$

This is the smallest length that can be measured with a screw gauge. This is its **least count**. Since the least count of a typical screw gauge is 0.01 mm or 10 micrometer, it is often called a micrometer screw gauge.

c) Zero Error

Similar to the vernier callipers, screw gauge can also suffer from a zero error. When the anvil end *A* and spindle end *B* touch each other but the zeros of the circular and main scales do not coincide, the screw gauge is said to have **zero error**. When the ends *A* and *B* touch each other and the zeros of the main scale and circular scale coincide, the screw gauge is said to have **no zero error** (Fig.1.7a). The zero error is said to be **positive** if the zero of the circular scale is below the zero of the main scale (Fig.1.7b). If the zero of the circular scale is above that of the main scale, the zero error is said to be **negative** (Fig.1.7c). As in the case of vernier callipers***, the zero error (with its sign) is always subtracted from the actual reading of the screw gauge.*** Hence, Final Reading = Reading Taken – Zero Error (with appropriate sign)

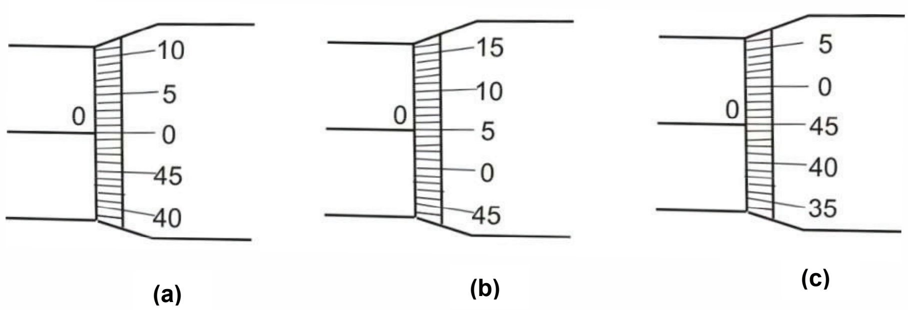
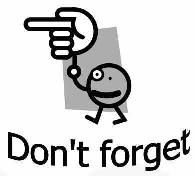


Fig.1.7: Relative positions of the zeros of linear scale and circular scale of a screw gauge when its spindle is in contact with anvil for a) no zero error; b) positive zero error; and c) negative zero error.

The screw gauge is a sensitive device and you should use it carefully.

While handling a screw gauge, you should take care of some important points:

- **Do not over-tighten the gauge; and**
- **Adjust the screw gauge to the point where it should read zero. In case it shows a different reading, note the error.**



We hope you can now confidently work with a screw gauge and take necessary precautions while using it to measure the length. For this part of the experiment you will need a screw gauge and a thin object like sheet of paper or a piece of wire or a pin.

1.3.2 Measurements using Screw Gauge

You should follow the steps listed below to measure the thickness of the given object:

1. Take a screw gauge and check whether or not its ratchet functions properly. If not, change the screw gauge.
2. Note the length of the smallest division on the linear (or main) scale and record it in Observation Table 1.2. Rotate the screw through ten complete rotations and note the distance advanced on the Main Scale on the screw. From this, you can calculate the distance by which the screw (that is spindle) moves in one complete rotation. This is the **pitch** of the screw. Note the total number of divisions on the circular scale (CS). By dividing the pitch of the screw by the total number of divisions on the circular scale, *N*, you will obtain the least count (LC). Usually, the LC of a screw gauge is 0.001 cm.
3. Touch the anvil with spindle and note the zero error, if there is one. Note it down with proper sign in the Observation Table 1.2.
4. Place the sheet/wire between the anvil and spindle. Tighten the screw so that the object is just held between them. Do not apply excess pressure to tighten the screw. You can apply optimum pressure for tightening by rotating the ratchet.

5. Note the readings on the linear and circular scale and record them in Observation Table 1.2.
6. Repeat steps 4 and 5 at least six times by taking the thickness measurements at different places. In this way, you can account for non-uniformity of the object thickness. Record all your observations in Observation Table 1.2.
7. Subtract the zero error (with its sign), if any, from each measured value. Calculate the mean value of the thickness of the given sheet/wire.
8. Calculate the average value and error and record your result as before.

Observation Table 1.2: Measurement of thickness

The length of the smallest division on the linear scale = ………………… mm

Distance advanced by the screw when it is given ten rotations = *D* = ………….. mm

Pitch of the screw = *P* (= *D*/10) = ……………….. mm

Number of divisions on the circular scale (*N*) = ………………..

Least count of the screw gauge $\textbf{LC} = \dfrac{\text{Pitch}}{N}$ **= …… mm = …… cm**

Zero error (with + or – sign) = ……… mm

Sl. No.	Linear scale reading LS (mm)	Circular scale reading × LC = CS (mm)	Thickness (mm)	
			Measured (=LS+CS)	Corrected (Measured value – Zero error)
1.				
2.				
3.				
4.				
5.				
6.				

Average thickness = ………. mm

Result: The thickness of the sheet is ……… cm ± ……….. cm

So far, you have learnt about devices used for length measurement with greater accuracy. In these cases, the object to be measured was always in contact with the measuring instrument. Now we will discuss about travelling microscope, which makes measurements in non-contact mode.

1.4 MEASUREMENT OF THE INTERNAL DIAMETER OF A CAPILLARY USING TRAVELLING MICROSCOPE

In various experiments you need to measure small lengths from a little distance away from the object. For example in the bending of beam apparatus, you need to measure a minute displacement of a metal beam caused by applied weights hanging to it. Also you need to precisely measure the distance between the bright and dark fringes formed by interference pattern in some optics experiments. For such measurements in non-contact mode, we make use of a travelling microscope. In this part of the experiment, you will be measuring the internal diameters of a glass capillary tube using it.

For this part of the experiment you will need a travelling microscope, spirit level, stand and a piece of capillary tube. But, before doing the experiment, you need to set the travelling microscope on your work table and understand how it works.

1.4.1 Working with Travelling Microscope

In Fig.1.8 you see a picture of a typical travelling microscope. It is basically a compound microscope which can be moved horizontally and vertically.

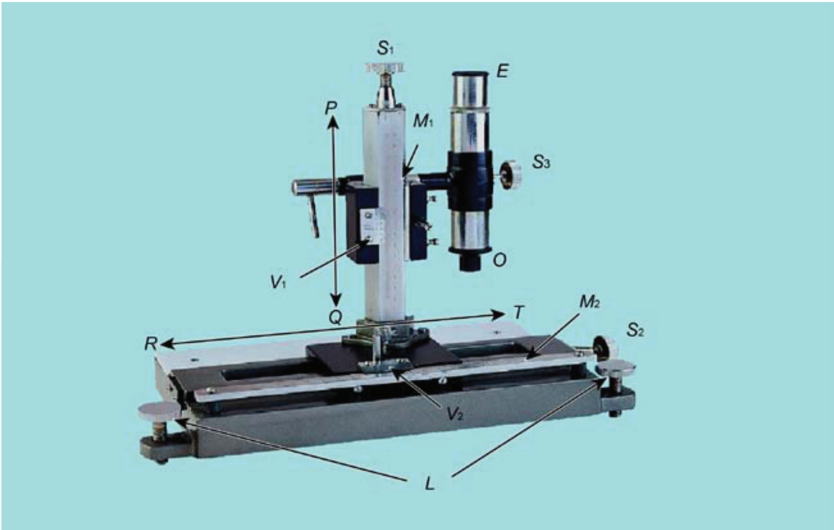


Fig.1.8: A travelling microscope.

It has two lenses: an eyepiece (*E*) and an objective (*O*). You can move it vertically along *PQ* using the screw S_1 and horizontally along *RT* using the screw S_2. The distance moved on the vertical scale is measured using the main scale M_1 and the associated vernier scale V_1. The distance moved on the horizontal scale is measured using the main scale M_2 and the associated vernier scale V_2. You should view the microscope through the eyepiece while the objective lens faces the object being viewed. You can focus the microscope with the help of the screw S_3 attached to its body.

1.4.2 Measurements using Travelling Microscope

Setting up the microscope

The eyepiece has a crosswire (shown as dotted lines in Fig. 1.9) which you may focus by sliding the eyepiece in or out. Now, to set the microscope, follow the steps given below:

- There are four levelling screws (*L*) on the base of the microscope (Fig.1.8). To start with you should ensure that the base of the travelling microscope is perfectly horizontal by using a spirit level and these screws.

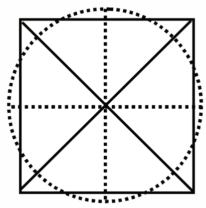


Fig.1.9: Coinciding the crosswire with a cross drawn on a piece of paper.

- Next, check that there is a free movement of the microscope in both the vertical and horizontal directions using the screws S_1 and S_2.
- Keep the microscope in the vertical position. Mark a cross on a small piece of paper with a pen (shown by solid lines in Fig.1.9) and place it below the objective.
- Gently move the eyepiece using the screw S_3 to focus on the cross, such that the crosswires of the eyepiece coinciding with centre of the cross and are clearly visible. In this condition, the microscope is said to be focussed.
- Yet another thing you need to do before doing the actual experiment is to calculate the least count of the two vernier scales V_1 and V_2. You can calculate it using the formula for the least count. (Usually both the vernier scales have the same least count.)

Now that you have set the microscope, you are ready to use it to measure the inner diameter of a capillary tube.

Fig. 1.10: Capillary tube held in a retort stand.

Procedure of Measurement

1. Hold the capillary tube horizontal in the clamp of a retort stand, as shown in Fig.1.10. Turn the microscope tube into horizontal position, such that the objective faces the capillary and focus it on the end of the capillary, which has a bore.
2. Adjust the travelling microscope to such a position that the vertical crosswire is exactly at the centre of the bore and horizontal crosswire just touches the bore at point *A* as shown in Fig. 1.11a. Note down the main scale and vernier scale reading on the vertical scale (M_1 and V_1) of the travelling microscope and enter the reading in the column *A* in Observation Table 1.3.

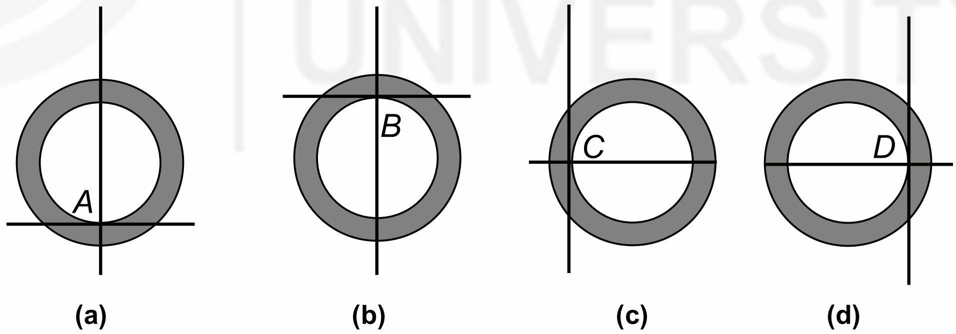


Fig.1.11: Different positions of the crosswire of the travelling microscope while measuring the diameter of a capillary tube.

3. Now using the screw S_1, move the travelling microscope in the vertical direction in such a way that the crosswire touches the point exactly opposite to *A* (the point *B* in Fig. 1.11b). Note down the main scale and vernier scale reading on the vertical scale of the microscope and enter the reading in the column marked *B* in the observation table.
4. Next, bring back the horizontal crosswire in the middle of the bore and move the travelling microscope in the horizontal direction using S_2 in such

a way that vertical crosswire touches the bore at point *C* as shown in Fig.1.11c. Note down the main scale and vernier scale reading on the horizontal scale (M_2 and V_2) of the microscope in this position and enter the reading in the column marked *C*.

5. Finally, move the travelling microscope in the horizontal direction in such a way that the vertical cross wire touches the bore at the point exactly opposite to *C* (point *D* in Fig. 1.11d). Note down the main scale and vernier scale reading on the horizontal scale of the microscope and enter the reading in the column marked *D*.
6. Using the above readings, you can calculate the inner diameter of the tube in vertical and horizontal directions. By calculating the mean of the vertical and horizontal readings, you can find the internal diameter of the tube.
7. Repeat steps 2-5 two more times to get 3 sets of readings and calculate the average diameter of the capillary tube.

Observations

Least count of vernier scale V_1:

$$LC_1 = \frac{\text{value of one } MSD \text{ on } M_1}{\text{number of divisions on the vernier scale } V_1} = \text{............ cm.}$$

Least count of V_2:

$$LC_2 = \frac{\text{value of one } MSD \text{ on } M_2}{\text{number of divisions on the vernier scale } V_2} = \text{............ cm}$$

Note that:
Reading = MS (main scale reading) +
VS (vernier scale reading) × *LC* (LC_1 or LC_2)

Observation Table 1.3: Microscope readings

Number of Observations	Microscope readings for crosswire in position				Internal diameter		
	A (cm)	*B* (cm)	*C* (cm)	*D* (cm)	Vertical $Y=B-A$ (cm)	Horizontal $X=D-C$ (cm)	Diameter $=(X+Y)/2$ (cm)
1.							d_1=
2.							d_2=
3.							d_3=

Average diameter= $\frac{d_1 + d_2 + d_3}{3}$ cm = ... cm.

EXPERIMENT 2

DETERMINATION OF MOMENT OF INERTIA OF A FLYWHEEL ABOUT ITS AXIS OF ROTATION

Structure

2.1 Introduction
Expected Skills
2.2 Theory of Flywheel
2.3 Procedure for Measuring Moment of Inertia

2.1 INTRODUCTION

In Experiment 1, you have learnt how to measured lengths of small orders using apparatus/ devices like vernier calliper and screw gauge. In some special cases, you make use of travelling microscope as well. Time and mass are other fundamental quantities in Physics. You will be learning how to measure mass in other experiments of this lab. In this experiment, you will get an opportunity to determine the moment of inertia of a flywheel. In this process you will be measuring diameter of the axle of the flywheel and time for the number of rotations that the flywheel makes before it comes to rest. We begin our discussion by presenting the general concept of moment of inertia and the moment of inertia of a flywheel in particular, for completeness. We shall then explain the procedure for doing the experiment.

Expected Skills

After performing this experiment, you should be able to:

- ❖ investigate how the moment of inertia depends on the mass suspended from the cord and the distance through which it falls; and
- ❖ determine the moment of inertia of a flywheel.

The apparatus required for this experiment is listed below.

Apparatus required

A flywheel, weight box (about 100 g, 200 g, 300 g etc.), stop watch, meter scale, a cord/string, vernier callipers.

2.2 THEORY OF FLYWHEEL

In Unit 12 of the theory course on Mechanics, you have learnt the concept of inertia. According to Newton's first law, every object continues to be in a state of rest or uniform motion in a straight line unless an external force acts on it. It means that every object offers resistance to change in its state of motion or rest. This resistance or inertness of bodies to change in motion or state of rest is called **inertia**. Inertia is directly proportional to the mass of the body. That is, in translational motion, mass is a measure of inertia. In rotational motion, where a body rotates about a fixed axis, the resistance offered depends not only on its mass but also on the distances of various parts of the body from the axis of rotation. The resistance of a body to change in rotational motion is called **moment of inertia (MI)**. So we can say that the moment of inertia is the rotational analogue of mass. You have learnt about it in Block 3 of the theory course on Mechanics. It has applications in drawing water from a well for irrigation where bullocks move in a circular path. In practice, a flywheel is fixed on the axle of moving parts of machines. It helps to steady the motion; absorb energy when a machine runs faster and supplies energy when it tends to slow down. Let us determine the expression of the MI of a flywheel.

Moment of inertia of a flywheel

Study Fig. 2.1. It shows a rigid body of mass M rotating about a fixed axis passing through O. Note that $M = \sum_i m_i$ where m_i is the mass of the i^{th} particle. All individual particles of the rigid body describe circular paths about the axis of rotation. Note that the linear velocity and distance from the axis of rotation are different for each particle but the angular velocity (ω) of all particles is the same.

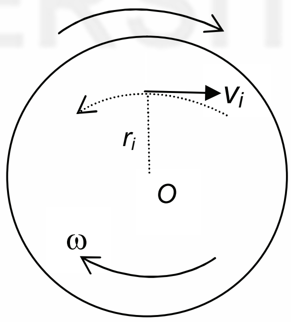


Fig. 2.1: A rigid body.

The kinetic energy of a particle situated at a distance r_i from the axis of rotation and moving with linear speed v_i is $\frac{1}{2}m_i v_i^2$. Since $v_i = r_i\omega$, we can write the expression of kinetic energy as $\frac{1}{2}m_i r_i^2\omega^2$. Thus, the expression of total kinetic energy (KE) of the body rotating about a fixed axis is

$$\text{KE} = \sum \frac{1}{2}m_i r_i^2\omega^2$$

If ω is constant, we can rewrite it as

$$\text{KE} = \frac{1}{2}\omega^2\sum_i m_i r_i^2 = \frac{1}{2}\omega^2 I$$

where I is the moment of inertia: $I = \sum m_i r_i^2$ of the body about the axis of rotation.

The moment of inertia of a body about a given axis is defined as the sum of the products $m_i r_i^2$ taken for all particles making up the body; m_i is the mass of the i^{th} particle, and r_i, its distance from the axis of rotation.

Note that the moment of inertia is not a constant quantity. It depends on the manner in which the mass is distributed about the axis of rotation. For example, the moment of inertia of a cylinder about its own axis is $\frac{1}{2}Mr^2$ (Fig. 2.2a) while its moment of inertia about an axis passing through its middle point and perpendicular to the cylinder's axis is $M\left(\frac{\ell^2}{12}+\frac{r^2}{4}\right)$ as shown in Fig. (2.2b).

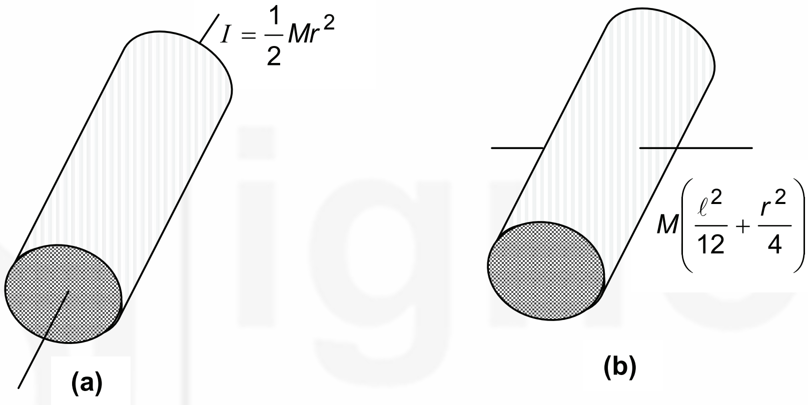


Fig. 2.2: Moment of inertia of a cylinder (a) about its own axis; (b) about an axis passing through its middle point and perpendicular to the cylinder's axis.

The flywheel

A flywheel is a heavy wheel with an axle as shown in Fig. 2.3. The mass of the flywheel is concentrated mostly in the rim. In your physics laboratory, you will notice that the wheel is set up in a wall with axle at a suitable height from the ground.

One end of a string is fixed to a small peg/pin on the axle and its other end carries a mass M. The string is completely wrapped around the axle. When the mass M is released, the string unwinds itself, thus setting the flywheel in rotation.

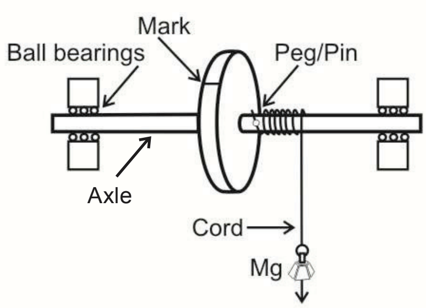


Fig. 2.3: A flywheel with an axle.

As the mass M falls, the rate of rotation of the flywheel increases till it becomes maximum when the string leaves the axle and the mass drops off. Suppose that the vertical height through which mass M drops before the string leaves the axle is h. So we can say that weight Mg, falls through a vertical height h and loses potential energy, PE (= Mgh). It is used in imparting linear velocity to the mass and the angular velocity to the flywheel. We denote these by v and ω, respectively, at the instant the mass drops off.

SAQ 1

What considerations lead us to choose an appropriate mass?

We can say that potential energy is used up in

i) imparting rotational kinetic energy $\left(\frac{1}{2}I\omega^2\right)$ to the flywheel, where I is the moment of inertia of the flywheel about the axis of rotation and ω, its angular speed,

ii) doing work against friction at the axle, and

iii) generating kinetic energy in the falling weight $\frac{1}{2}Mv^2$.

You will note that the flywheel continues to rotate after the weight is detached from the peg. Suppose it makes n rotations in time t before coming to rest. Then the average angular velocity of the flywheel is $\frac{2\pi n}{t}$. Assuming that the motion of the flywheel is uniformly retarded by the frictional force at the axle and once the final angular velocity is zero, its initial angular velocity must be (see margin remark)

$$\omega = \frac{4\pi n}{t} \tag{2.1}$$

Now, the kinetic energy of the flywheel is $\frac{1}{2}I\omega^2$ and this is dissipated in n rotations of the wheel. The energy lost per rotation in overcoming friction is $\frac{1}{2}\frac{I\omega^2}{n}$.

If at the start of the motion the string was wrapped n_1 times round the axle, the potential energy of the falling weight used up in overcoming friction is $n_1\left(\frac{1}{2}\frac{I\omega^2}{n}\right)$. Also if v is the velocity of the falling weight at the moment it leaves the peg, its kinetic energy is given by $\frac{1}{2}Mv^2$. Hence, we can write

$$Mgh = \frac{1}{2}I\omega^2 + \frac{n_1}{n}\frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 \tag{2.2}$$

On substituting $v = r\omega$, we can rewrite Eq. (2.2) as

After the mass detaches, its angular velocity decreases on account of friction and after some time t, the flywheel finally comes to rest. At the time of detachment of the mass, the angular velocity of the wheel is ω and when it comes to rest its angular velocity is zero. Hence, if the force of friction is steady, the motion of the flywheel is uniformly retarded and the average angular velocity during this interval is equal to $\omega/2$. Thus

$\omega/2 = 2\pi n/t \Rightarrow \omega = 4\pi n/t$

$$Mgh = \frac{1}{2} I\omega^2 \left(1 + \frac{n_1}{n}\right) + \frac{1}{2} Mr^2\omega^2$$

where r is the radius of the axle.

On rearranging the terms of the above expression, we get

$$I = \frac{\frac{2Mgh}{\omega^2} - Mr^2}{\left(1 + \frac{n_1}{n}\right)} = \frac{2Mgh}{\omega^2\left(1 + \frac{n_1}{n}\right)} - \frac{Mr^2}{\left(1 + \frac{n_1}{n}\right)} \quad (2.3)$$

Thus by observing time t and taking the values of n and n_1 for the flywheel, the moment of inertia can be calculated using Eq. (2.3).

2.3 PROCEDURE FOR MEASURING MOMENT OF INERTIA

Follow the steps given below:

1. Take a vernier callipers and determine its least count. You have learnt this in Experiment 1.
   - Identify the main scale (MS) and the vernier scale (VS) on the callipers and write the number of divisions on VS:
     No. of divisions in the vernier scale = ……………………………………
   - Note how many of MS divisions equal all VS divisions:
     ... No. of divisions in the main scale = .. No. of divisions on vernier scale
   - Calculate the least count of your vernier callipers.

   Measure the diameter of the axle in two mutually perpendicular directions as shown in Fig. 2.4 at a number of positions. Record your observations taking care of zero error (if any) in the Observation Table 2.1. Determine its mean value. Note that a small error in the value of r will influence the result adversely because the expression of I contains r^2 term.

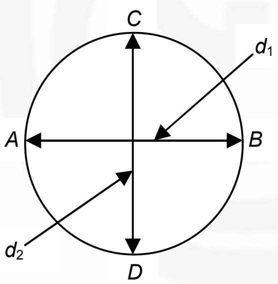


Fig. 2.4: Axle of the flywheel.

2. Take a string whose length is less than the height of the axle from the floor.
3. Make a loop at one end of the string and put it round the peg (a brass pin fitted on the axle of the flywheel as shown in Fig. 2.3).
4. Rotate the wheel anticlockwise with hand and wrap the string evenly and uniformly around the axle. Make sure that there is no overlapping of or gaps between various loops. When almost the whole string has been wound, mark on the string where its contact with the axle just ceases. Count the number of turns wound and note them under n_1 in Observation Table 2.2. It is expected that in your experiment value of n_1 will be constant.
5. Attach a mass say, 100 g on the free end of the string.
6. When the mass is just below the rim, make a reference mark on the wall.

7. Release the mass. You will note that the string gets unwrapped and the flywheel moves in clockwise direction.
8. If you are working in pairs, one of you should start counting the number of rotations of the flywheel and the other one should start the stop watch at the moment the mass detaches. Count the number of rotations (n) that the flywheel makes before it comes to rest. Stop the stop-watch just when the flywheel comes to rest.
9. Record the time for which the flywheel continues to rotate after the detachment of the mass (t).
10. Measure the length of the string between the loop and the mark at the other end, h, the distance descended by the mass. Record the readings in Observation Table 2.2.
11. Repeat steps 4-10 by making the flywheel rotate in the anticlockwise direction.
12. Repeat steps 4-11 by attaching different masses on the string.

Observation Table 2.1: Measurement of the diameter of the axle of the flywheel

Least count of the vernier callipers = ……………………… cm
Zero error (With proper sign + or –) = ……………………… cm

S.No.	Reading along *AB* d_1(cm)	Reading along *CD* d_2(cm)	$(d_1+d_2)/2$ (cm)
1.			
2.			
3.			
4.			
5.			

Mean diameter (d) =………………………cm
(% error)=……………………
Mean radius of the axle $r = d/2$ = ………±…. cm

Observation Table 2.2: Measurement of h, n, n_1 and t

Least count of stop-watch = ……….s

Set No.	Mass M(g)	Rotation	Height h(cm)	n_1	n		t(s)	
					Measured	Average	Measured	Average
1	100	Clock-wise						
		Anti-clockwise						
2	200	Clockwise						
		Anti-clockwise						

Calculations:

The moment of inertia of the flywheel can be obtained using Eq. (2.3). In this equation, the value of ω is substituted. Hence, the expression can be rewritten as

$$I = \frac{2Mgh}{\frac{(4\pi n)^2}{t^2}\left(1+\frac{n_1}{n}\right)} - \frac{Mr^2}{\left(1+\frac{n_1}{n}\right)}$$

Calculate the values of moment of inertia (I) for all the sets separately using the above formula. Report your result by taking the mean value of I.

Result:

Mean value of moment of inertia (I) of the flywheel about its axis is

= ……….g cm^2 = ………kg m^2

Precautions for Minimization of Error

1. The string should be uniformly and evenly wound on the axle.
2. The stop-watch should be started at the instant when the string leaves the peg.
3. The loop slipped over the peg on the horizontal axle should be loose so that when the string has unwound itself completely, there is no tendency for the string to rewind in the opposite direction.
4. The axle should be oiled well to minimize friction.

SAQ 2

How does moment of inertia depend upon the axis of rotation?

EXPERIMENT 3

DETERMINATION OF YOUNG'S MODULUS BY BENDING OF BEAMS

Structure

3.1 Introduction
  Expected Skills
3.2 Theory of Cantilever
  Working Principle of a Cantilever
  Bending Moment
  Depression at the Free End of a Cantilever
3.3 Procedure for Measuring Depression in a Beam using a Microscope
3.4 Measurement of Depression in a Beam using a Telescope and an Optical Lever
3.5 Comparison of Accuracies

3.1 INTRODUCTION

In the previous experiment, you have learnt about the concept of moment of inertia and measured the moment of inertia of a flywheel. In some experiments of this course you will measure certain elastic properties of matter about which you have studied in school physics. In this experiment you will learn to determine Young's modulus of a material by the method of bending of beams. Young's modulus is an indicator of how elastic a one-dimensional object is when a force is exerted along its length. In the Sec. 3.2, we briefly describe the underlying theory. In Sec. 3.3, we describe the procedure for measuring depression in a beam using a microscope. In Sec. 3.4, you will learn how to perform the experiment and take measurement of depression in a beam using a telescope and an optical lever. Finally in Sec. 3.5, you will compare the accuracies in measurement of depression using microscope and telescope methods.

In the next experiment, you will learn to measure the modulus of rigidity of a wire using Maxwell's needle.

Expected Skills

After performing this experiment, you should be able to:

- focus a microscope and a telescope on a given object;
- remove parallax error;

- measure small depressions;
- measure depression of the beam using (i) a microscope and (ii) a telescope and optical lever arrangement and compare the accuracies of the two methods; and
- calculate the value of Young's modulus of elasticity.

The apparatus required for this experiment is given below.

Apparatus required

A rectangular steel beam, two knife-edges, a travelling microscope, a pin, an optical lever and scale arrangement, a telescope, metre scale, a hanger for hanging weights in the middle of the beam, a set of half-kilogram weights, vernier callipers and a screw gauge.

Let us now describe the underlying theory of this experiment.

3.2 THEORY OF CANTILEVER

If you press a rubber ball or a piece of sponge, you will observe that their shape undergoes a change. What happens when you stop pressing them? You will observe that they regain their original shape. In fact, all bodies can, more or less, be deformed by a suitably applied force and when the deforming force is removed, they tend to recover their original state. The simplest case of deformation is observed when we stretch a wire fixed at one end. Addition of further weight at its other end increases its length. When the suspended weight is removed from the wire, it tends to come back to its original length. You may similarly have observed that a train running over a bridge produces a depression in the rails. However, they attain their normal state once the train has passed. It means that a body opposes any change in its shape and/or size by an external force. And once the external force is removed, the body tends to regain its original normal state. This property is called **elasticity**. Greater the force necessary to produce deformation in the body, more elastic it is said to be.

The maximum stress a material can sustain without undergoing permanent deformation is termed as its **elastic limit.**

When a body is subjected to a deforming force, an opposing force comes into play and tends to resist the effect of applied force. In equilibrium state, the restoring force is equal to the applied external force. The restoring force per unit area set up inside the body is called **stress**. The fractional change in the length, volume or shape of the body is termed as **strain**. For example, when a wire is stretched by applying a force along its length, i.e., normal to its cross-sectional area, the change occurs in its length. The change in length per unit original length of the wire is called **longitudinal strain**. The ratio of stress to longitudinal strain, within the elastic limit, is called **Young's modulus**. The value of Young's modulus depends on the nature of the material rather than the physical dimensions of the sample.

The knowledge of Young's modulus is vital for bridge design as we need to know the precise deformation (depression) in a loaded structure and its parts. Refer to Fig. 3.1. When a train passes over the bridge, the beam bends. Its upper surface is compressed whereas the lower surface is stretched. These deformations are transmitted to other parts of the bridge also. Young's modulus also enables us to know the stress which a body, say the connecting rod or piston of a steam engine or a girder, can bear. (You must have observed the girders and beams used in bridges and high rise buildings. The girders are manufactured with their cross-section in the form of the letter I. In a beam of rectangular cross-section, the longer side is used as the depth.) In this experiment you will learn to determine the Young's Modulus using bending of the beam.

A **beam** is a bar of uniform cross-section (circular or rectangular) of a homogeneous, isotropic (same properties at all points and in all directions) elastic material.

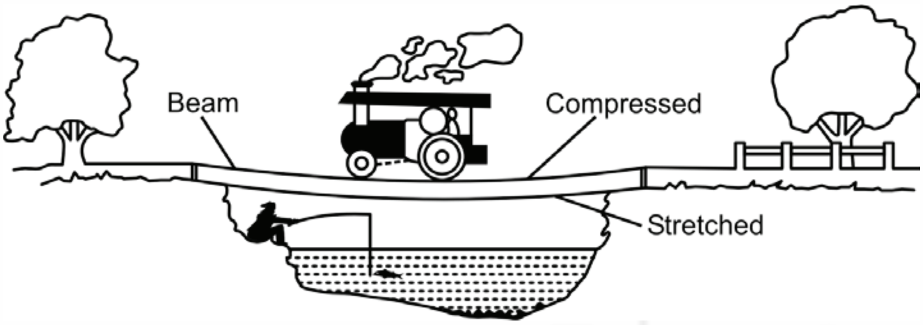


Fig. 3.1: A railway engine moving over a railway bridge produces depression in the beam.

When a beam is supported near its ends and loaded at the centre, it shows maximum depression at the loaded point. However, the depression produced in the beam depends on its material; in a steel beam, it is so small that you cannot observe it with unaided eye. Refer to Fig. 3.2, which shows a beam supported on two knife-edges indicated by *A* and *B*. Suppose that it is loaded in the middle at *C* with a weight *W*. The reaction at each knife edge can be taken to be (*W*/2) in the upward direction. In this position, the beam may be considered as equivalent to two inverted cantilevers (read the margin remark), fixed at *C*. The bending in these two cantilevers will be produced by the reaction load – acting upwards at *A* and *B*. Therefore, it is important for us to know how the bending is produced in a cantilever and on what factors it depends.

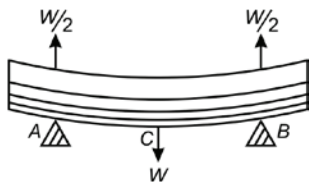


Fig. 3.2: A beam supported near the two ends and loaded at the centre.

A cantilever is a beam fixed horizontally at one end.

3.2.1 Working Principle of a Cantilever

Consider the cantilever shown in Fig. 3.3a. Suppose that, weight W_1 is acting at the free end. As soon as the beam is loaded, it bends. Do you know why? To discover answer to this question, consider the section P_1QRP_2 of the beam. Since the load W_1 has been applied at the free end of the beam, the restoring force acts vertically upward along $P_2\,P_1$. These two forces, the load and the restoring force, being equal and opposite, form a couple. You will recall from your school physics and Unit 12 of Block 3 of the theory course on Mechanics that a couple has the tendency to rotate a body. However, a cantilever cannot rotate because it is fixed at one end. Therefore, the beam bends in the clockwise direction. This is indicated by the arrow. For this reason, this couple is called **bending couple** and the moment of this couple is called **bending moment**.

You may now ask: How can a beam be in equilibrium when a couple acts? This can happen when a balancing couple is acting on the beam. To know how balancing couple is formed, let us understand what changes take place in the interior of the beam when its free end is loaded. For this purpose, imagine the beam to be made up of a large number of small elements placed one above the other. These small elements are called **filaments**. When a cantilever is loaded, the filaments in the upper-half of the beam are stretched and the filaments in the lower-half are compressed. However, a surface (or filament) exists in the middle, which is neither stretched nor compressed. This surface, known as **neutral surface,** is denoted by *EF* in Fig. 3.3b.

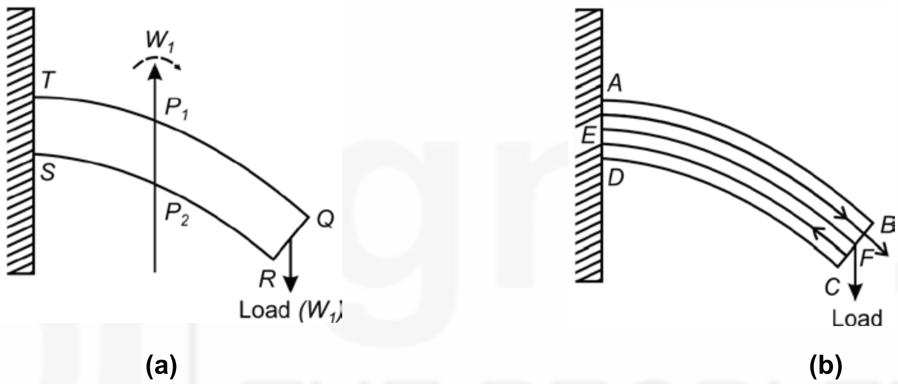


Fig.3.3: a) When a cantilever is loaded, it bends; b) filaments in the interior of a cantilever under the action of a bending couple.

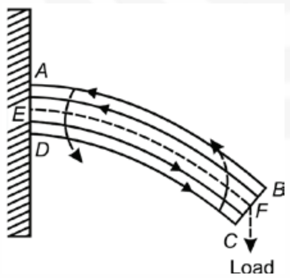


Fig. 3.4: The moments of forces about the neutral axis indicated by dashed line oppose bending.

Due to changes induced by the couple, restoring forces are developed in the filaments, as shown in Fig. 3.4. Above the neutral surface, these forces act towards the fixed end of the beam and tend to oppose extension. On the other hand, below the neutral surface, restoring forces act towards the loaded end and oppose further compression. These two sets of forces act in opposite directions and their moments about the neutral surface are directed in the anticlockwise direction (indicated by dotted arrows). This direction is opposite to that in which the beam has been bent due to the bending couple. This set of forces constitutes balancing couple and tends to restore the beam to its original condition. When the beam is in equilibrium, the moment of couple is equal to the bending moment. You may now like to know the factors on which the bending moment depends.

3.2.2 Bending Moment

Consider a small portion of the beam shown in Fig. 3.5a. It is bent in the form of an arc. Suppose that an element *ab* on the neutral surface subtends an angle θ at the centre of curvature. Also let *R* be the radius of curvature of the part a_0b_0 of the neutral surface. Then the length of portion $a'b'$ of a filament, which is at a distance z from the neutral surface (filament), can be expressed as $a'b' = (R + z)\,\theta$.

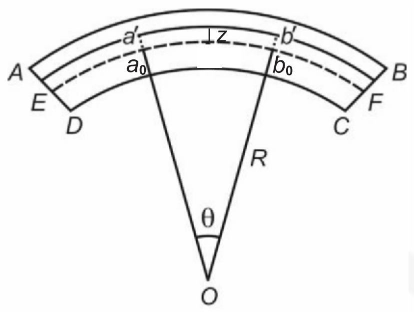
When the beam is not bent, the length of this filament is equal to the length $R\theta$ of the neutral filament. Therefore, increase in length can be written as

$$a'b' - a_0b_0 = (R + z)\,\theta - R\,\theta = z\,\theta \qquad (3.1)$$

Hence

$$\text{Longitudinal strain} = \frac{\text{Increase in length}}{\text{Original length}} = \frac{z\theta}{R\theta} = \frac{z}{R} \qquad (3.2)$$

Now consider a section $LMNT$, which is perpendicular to the length of the beam and its plane of bending, as shown in Fig. 3.5b. In this section, consider a small element of area a at a distance z from the neutral surface. The strain produced in the filament passing through this area will be z/R.



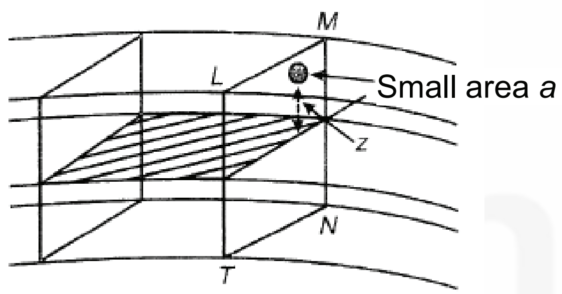


Fig. 3.5: a) A small portion of the beam in strained condition; and b) *LMNT* is the cross-section of the beam perpendicular to its length and the plane of bending.

From the preceding sub-section, you may recall that whenever the length of a filament increases, a force acts on the filament towards the fixed end of the beam. You can calculate the magnitude of this force by noting that

$$Y = \frac{\text{Stress}}{\text{Longitudinal strain}}$$

so that stress is a product of Young's modulus of the material of the beam and longitudinal strain. This shows that stress on area a is

$$S = Y\frac{z}{R} \qquad (3.3)$$

And the magnitude of force acting on area a is given by

$$F = \text{Area} \times \text{Stress} = aY\frac{z}{R} \qquad (3.4)$$

Moment of this force about the neutral surface is equal to the product of force and its distance from the neutral surface:

$$M = Ya\frac{z}{R}z = Ya\frac{z^2}{R} \qquad (3.5)$$

The total moment of the forces acting on all the filaments in the section $LMNT$ (or in the beam) is given by:

$\sum az^2$ is moment of inertia, I_g of the beam about the neutral surface. Therefore, it is equal to AK^2, where A is area of cross section of the beam and K is its radius of gyration about the neutral surface. For a rectangular cross-section, $A = b \times d$ and $K^2 = \frac{d^2}{12}$, where b is length and d is width of the rectangular portion.

$$\therefore \; I_g = AK^2 = \frac{bd^3}{12} \qquad \text{(i)}$$

For a circular cross-section, $A = \pi r^2$ and $K^2 = \frac{r^2}{4}$ where r is its radius.

$$\therefore I_g = AK^2 = \frac{\pi r^4}{4} \qquad \text{(ii)}$$

$$\sum \frac{Yaz^2}{R} = \frac{Y}{R} \sum az^2 = \frac{Y}{R} I_g \qquad (3.6)$$

where $I_g = \sum az^2$ is the moment of inertia of the beam. Thus, the bending moment of the beam is given by $\frac{Y}{R} I_g$.

You may now like to know the relation between moment of the restoring couple and the depression at the free end of the cantilever.

3.2.3 Depression at the Free End of a Cantilever

Refer to Fig. 3.6. It shows a cantilever of length ℓ loaded at the free end. AB represents its neutral axis. Let us choose the x-axis along its length and the y-axis vertically downwards. When the free end of the cantilever is loaded with a load W_1, the maximum depression occurs at its free end. The neutral axis takes new position AB' and the end B is depressed by δ. Consider a section P of the beam at a distance x from end A. Due to the load W_1, the bending moment acting on this section is given by

$$W_1 \times PB = W_1 (\ell - x)$$

Since the beam is in equilibrium, this must be equal to $\frac{YI_g}{R}$, the moment of resistance to bending. Thus, we can write

$$W_1(\ell - x) = \frac{YI_g}{R} \qquad (3.7)$$

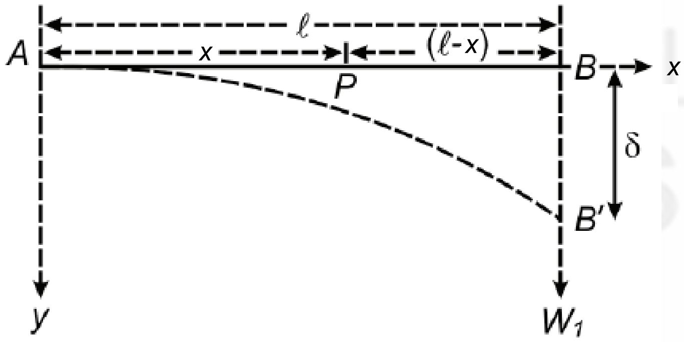


Fig. 3.6: A cantilever of length ℓ loaded at the free end.

Refer to any elementary book on differential calculus. The complete expression for radius of curvature is given by

$$\frac{1}{R} = \frac{(d^2y/dx^2)}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}$$

For small bending, $\frac{dy}{dx} << 1$ and we can ignore it in comparison to one in the denominator of above expression. This leads us to simple expression.

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Since the neutral surface remains unstretched, its radius of curvature (R) at any given point is given by the relation $\frac{1}{R} = \frac{d^2y}{d\,x^2}$ (read the margin remark).

Substituting this value of R in Eq. (3.7), we get

$$W_1(\ell - x) = YI_g \frac{d^2y}{d\,x^2}$$

or

$$\frac{d^2y}{d\,x^2} = \frac{W_1}{YI_g}(\ell - x) \qquad (3.8)$$

Integrating Eq. (3.8) twice with respect to x (read the margin remark), we get the value of depression (δ) at the free end:

$$\delta = \frac{W_1 \ell^3}{3YI_g} \quad (3.9)$$

Thus the depression at the free end of the cantilever is $\frac{W_1 \ell^3}{3YI_g}$.

SAQ 1 – Depression in a cantilever

By looking at Eq. (3.9), list the factors on which the depression at the free end of a cantilever depends.

Now refer again to Fig. 3.2. If the length of the beam *AB* is *L*, the length of both cantilevers (*AC* or *BC*) will be *L*/2. Since the reaction at each knife-edge is $W/2$, we can assume that each cantilever (*AC* or *BC*) is loaded at the free end by a load *W*/2. Then Eq. (3.9) can be used to calculate elevation Δ of *A* or *B* above *C* by substituting $W_1 = W/2$ and $\ell = L/2$:

$$\Delta = \frac{\frac{W}{2}\left(\frac{L}{2}\right)^3}{3YI_g} = \frac{WL^3}{48YI_g}$$

The elevation of *A* or *B* above *C* is the same as the depression of *C* below *A* and *B*. Therefore, on rearranging the above result, you can write

$$Y = \frac{WL^3}{48\Delta I_g}$$

For a beam with a rectangular cross section of width *b* and depth *d*, $I_g = bd^3/12$. Hence, in terms of the dimensions of the beam, the expression for Young's modulus simplifies to

$$Y = \frac{WL^3}{4\Delta bd^3} \quad (3.10)$$

From this result, you will note that to determine Young's modulus, you have to measure the depression at the centre of the beam when it is loaded with a known weight *W*. For steel bars, the magnitude of depression is very small, and has to be measured very accurately. For this purpose, you will learn how to use a microscope, telescope and the optical lever arrangement. Let us now describe the procedure for measuring Δ.

Integrating Eq. (3.8) with respect to *x* we get

$$\frac{dy}{dx} = \frac{W_1}{YI_g}\left(\ell x - \frac{x^2}{2}\right) + C_1$$

where C_1 is constant of integration.

When $x = 0$, $\frac{dy}{dx} = 0$

Hence $C_1 = 0$

$$\therefore \frac{dy}{dx} = \frac{W_1}{YI_g}\left(\ell x - \frac{x^2}{2}\right)$$

At the free end of the beam ($x = \ell$), $y = \delta$. Hence again integrating, between the applicable limits we have

$$\int_0^\delta dy = \frac{W_1}{YI_g}\int_0^\ell \left(\ell x - \frac{x^2}{2}\right) dx$$

$$\therefore \delta = \frac{W_1}{YI_g}\left(\frac{\ell^3}{2} - \frac{\ell^3}{6}\right) = \frac{W_1 \ell^3}{3YI_g}$$

3.3 PROCEDURE FOR MEASURING DEPRESSION IN A BEAM USING A MICROSCOPE

Follow the steps given below to measure the bending of beam by a microscope:

1. Place the given beam horizontally on the knife-edges, as shown in Fig. 3.7. See that equal (but small) portions of the beam project beyond the knife-edges and the smaller side of its cross-section is vertical.

2. Suspend a hanger for loading the beam, exactly at the centre, between the two knife-edges. Attach a small pin (vertically) at the centre of the beam with the help of wax for reading the position of the beam as shown in the figure.

3. Focus the travelling microscope on the pin and coincide its horizontal cross-wire with the tip of the pin. If you are not able to focus the microscope on the pin, you should seek the help of your counsellor.

4. Before you start taking observations, you should calculate the least count of the vernier callipers of the travelling microscope. For this purpose, note the value of the smallest division of the main scale of the microscope and the number of divisions on the vernier scale. The difference between the value of one smallest division of the main scale and value of one division of vernier scale gives its **least count**. Once you have focused the tip of the pin and coincided it with the horizontal cross-wire, you are ready to perform your experiment.

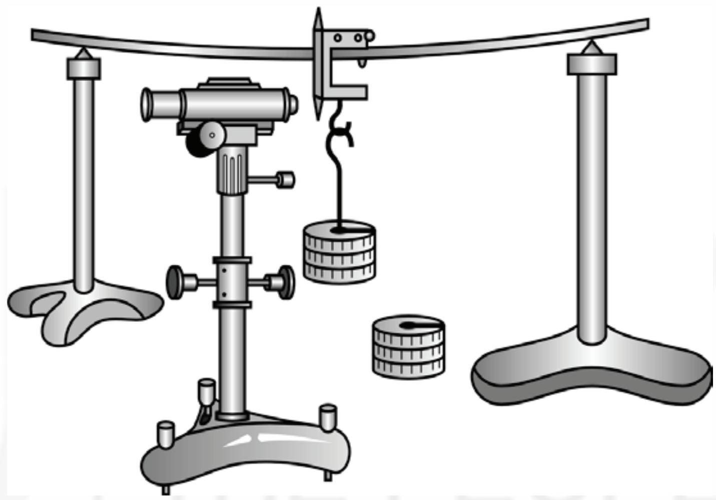


Fig. 3.7: Experimental arrangement for measuring depression of the beam using a microscope.

5. Read the main scale and the vernier scale readings. This is the reading when no load is placed in the hanger. Record it in Observation Table 3.1.

6. Next, without disturbing anything at all, place a weight of half-a-kilogram in the hanger. Is the tip of the pin visible in the field of view of the microscope? If so, does the tip of the pin coincide with the horizontal cross-wire? We expect that the tip will not coincide with the horizontal cross-wire because the beam has been depressed at the centre. You should observe that a gap appears between the tip of the pin and the horizontal cross-wire. Move the microscope vertically and make the tip of the pin to again coincide with the horizontal cross-wire of the microscope. Note the main scale and the vernier scale readings. Record these in Observation Table 3.1.

   Increase the load in equal steps of half-a-kilogram. Note the position of the pin by coinciding it with the horizontal cross-wire every time.

7. Now remove the weights gently in the same steps and note the microscope readings again.

8. Repeat step 7 till there is no weight on the hanger. Note that **the weight should be placed or removed from the hanger very gently**.

SAQ 2 – Elastic limit

Why is it necessary to take reading with decreasing load as well?

Observation Table 3.1: Measurement of depression using a microscope

Value of 1 small division of the main scale of the microscope (x) = …. cm
No. of vernier scale divisions (n) = …...
Least count of the microscope (x/n) = ….cm

Sl. No.	Load placed on the hanger W (g)	Microscope reading when the tip of the pin coincides with the horizontal cross-wire			Depression Δ (cm)
		with load increasing (cm)	with load decreasing (cm)	Mean (cm)	
1.	0				
2.	500				
3.	1,000				
4.	1,500				
5.	2,000				
6.	2,500				
7.	3,000				
8.	3,500				

This will give you two readings for each load: one with load increasing and the other with load decreasing. Calculate the mean of these two readings for a given load. Calculate the depression produced in the beam for each load by subtracting the initial mean reading from the mean reading for that particular load.

Plot a graph between the load (along x-axis) and depression (along y-axis). We expect the plot to be a straight line. Draw the best straight line passing as closely as possible through the observed points, as shown in Fig. 3.8. Calculate the slope of the straight line by choosing two widely separated points. The slope will give you the value of Δ/W.

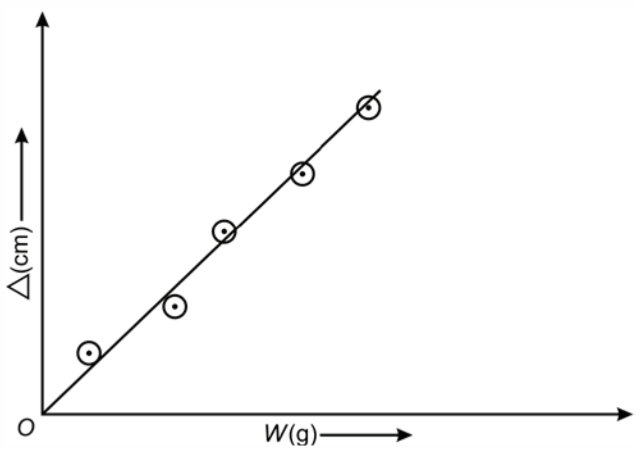


Fig. 3.8: Graph between depression (Δ) and load (W).

3.4 MEASUREMENT OF DEPRESSION IN A BEAM USING A TELESCOPE AND AN OPTICAL LEVER

To measure depression in a beam using a telescope, you will require an optical lever and scale arrangement. (An optical lever consists of a plane mirror mounted on a tripod stand.) To set up the apparatus follow the steps below.

1. Place the beam as in Step 1 of Sec. 3.3 of this experiment.
2. Remove the vertical pin and replace it by an optical lever such that the two legs supporting the mirror *M* rest on the fixed horizontal base *F* behind the beam and the third leg *L* rests on the beam at its centre *C*, as shown in Fig. 3.9. What will happen if you place the two legs supporting the mirror on the beam and the third leg on the base? If you do so, the depression will not correspond to the one at the centre. It is important to adjust the mirror so that it is vertical and parallel to the length of the beam.

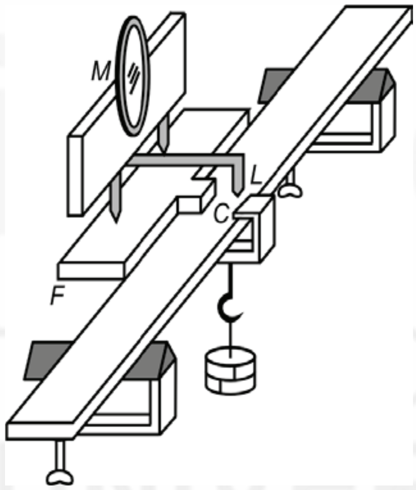


Fig. 3.9: Experimental arrangement for measuring depression of a beam using a telescope and an optical lever.

When a load is placed on the hanger, depression is produced in the beam. As a result, the leg of the optical lever touching the centre of the beam would go down. This would tilt the mirror forward. So, to know the depression in this part of the experiment, you have to measure the angle through which the mirror tilts. This requires the use of a telescope and a scale arrangement.

3. Fix a vertical scale in front of the mirror at a distance of about one metre on a rigid stand so that its image is visible in the mirror. Place the telescope close to the scale and at the same height as the mirror. Focus the eye piece so that the horizontal cross-wire of the telescope is distinctly visible. Now focus the telescope on the image of the scale in the mirror. For focusing this, you may have to turn the mirror slightly about its horizontal axis. If you are not able to focus the image of the scale clearly,

you should not waste time. Seek guidance of your counsellor and you should practice it a few times on your own thereafter. When you can clearly see the image of a scale in the mirror through the telescope, note the position of the horizontal cross-wire on the image of the scale and record it in Observation Table 3.2.

What does the position of the horizontal cross-wire signify? Refer to Fig. 3.10. Here M_1 is the initial position of the plane mirror. This means that what you have recorded is in fact division *A* of the scale.

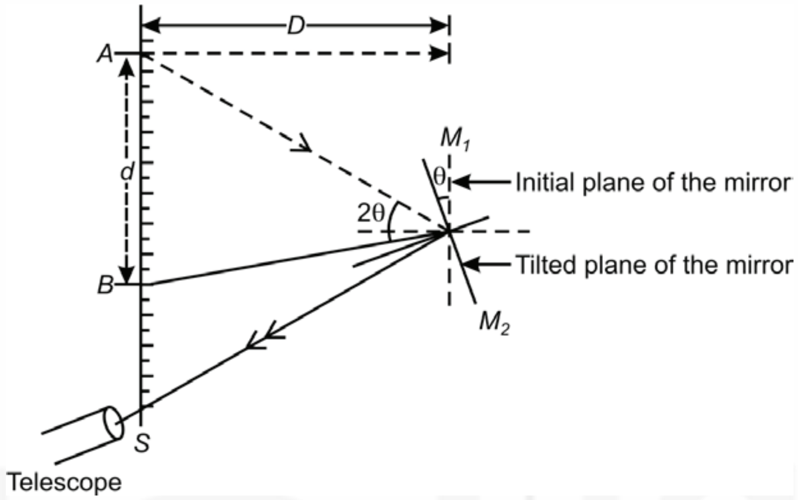


Fig. 3.10: Illustrating the principle of optical lever.

> We know that when a beam of light is incident on a plane mirror, which is turned through an angle θ about a vertical axis in its plane, the reflected ray turns through twice the angle.

4. Now gently place a load of 500 g on the hanger. This would depress the beam slightly. As a result of this, the mirror will tilt forward through an angle, say θ. Now, Instead of division *A* of the scale, you will see another division on the scale, say *B* (see Fig. 3.10) in the telescope after reflection from the plane mirror. Record its position in Observation Table 3.2.

5. Increase the load on the hanger in equal steps of half kg. Note down the position of the horizontal cross-wire of the telescope on the image of the scale after each addition of load.

6. Now decrease the load on the hanger in the same steps and note the position of the horizontal cross-wire on the image of the scale in the mirror every time. Record it in Observation Table 3.2. For each load, calculate the mean values of the two readings – one taken while increasing the load and the other while decreasing the load – of the cross-wire thus obtained. Calculate *d* for each load by subtracting the initial mean reading (d_0) from the mean reading for that particular load.

7. If distance between the two divisions *A* and *B* on the scale is *d* and *D* is the distance between the mirror and scale, then

$$2\theta = \frac{d}{D}$$

If the third leg is at a perpendicular distance *x* from the hind legs *P* and *Q*, the depression, Δ, of the beam for the given load is given by

$$\Delta = x\theta = \frac{xd}{2D} \qquad (3.11)$$

From this relation we find that once x, d and D are known, Δ can be readily computed.

8. Measure the distance D between the mirror and the scale and write it in Observation Table 3.2. To measure x, you should place the optical lever on a sheet of paper and press it lightly so that impressions of its feet are obtained on it. From these impressions, determine the perpendicular distance of the front foot of the optical lever from the line joining the hind legs. Using Eq. (3.11) you can readily know the depression (Δ) of the beam for each load and record it in Observation Table 3.2.

Observation Table 3.2: Measurement of depression using a telescope and an optical lever

Distance D of the scale from the mirror of optical lever =……………………. cm

Perpendicular distance x of the front foot of the optical lever from the line joining the other two legs =……………………. cm

Sl. No.	Load (W) placed on the hanger (g)	Position of the horizontal cross-wire of the telescope on the image of the scale (cm)			$d = s - d_0$ (cm)	$\Delta = \frac{xd}{2D}$ (cm)
		with increasing load	with decreasing load	Mean (s)		
1.	0			d_0 =…		
2.	500					
3.	1,000					
4.	1,500					
5.	2,000					
6.	2,500					
7.	3,000					
8.	3,500					

Plot a graph between load (W) along the x-axis and depression (Δ) along the y-axis. You should preferably use the same scale as you have used in case of W–Δ graph for a microscope. Calculate the slope of the straight line thus obtained. We expect it to be same as that obtained in Sec. 3.3.

3.5 COMPARISON OF ACCURACIES

Now, you have to calculate Young's modulus in both cases. To do so, you should measure the thickness and width of the beam and its length between the knife edges. To measure the length of the beam between the knife edges, you can use a metre scale. Using different parts of the scale, repeat the measurement several times and get the mean value. Record your readings in Observation Table 3.3(a).

Observation Table 3.3(a): Length (L) of the beam between knife-edges *A* and *B*.

Sl. No.	Scale reading for the knife-edge *A* x_1 (cm)	Scale reading for the knife-edge *B* x_2 (cm)	Length ($x_2 - x_1$) cm
1. 2. 3. 4. . .			

Mean length L (cm) = …………… $\pm$ …….

Use a screw gauge to measure the thickness of the beam at several places along its length. Make your own Observation Table 3.3(b) similar to the Observation Table 1.2 described in Experiment 1 and calculate the mean thickness.

Observation Table 3.3(b): Measurement of thickness (d) of the beam using screw gauge

Least count of the screw gauge =……...cm

Zero error (if any) with proper sign =……...cm

Mean thickness = …………cm

Corrected value (if zero correction is made) = …………cm

Take at least four readings to measure the width of the beam with vernier callipers at several places. Record the readings in Observation Table 3.3(c) prepared by you based on Observation Table 1.1 in Experiment 1. Calculate the mean value.

Observation Table 3.3(c): Measurement of width (b) of the beam using vernier callipers

Least count of the vernier callipers = ……..cm

Zero error (if any) with proper sign =……...cm

Mean width = ……….cm $\pm$ ……………… cm

SAQ 3 – Measurement of small lengths

Suppose that a good screw gauge or vernier callipers is not available in your lab to measure *d* and *b*. Which device – a metre scale, a microscope or the telescope will you use or recommend? Justify your answer.

Knowing *L*, *b*, *d* and the slopes of the straight lines obtained using a microscope and a telescope, you can easily calculate Young's modulus of the material of the beam using Eq. (3.10):

$$Y = \frac{L^3}{4bd^3} \times \frac{1}{\text{slope}} = \ldots\ldots\ldots\ldots\text{ dynes cm}^{-2}$$

$$= \ldots\ldots\ldots\ldots\text{ N m}^{-2}$$

Result: Young's modulus of the material of the given beam using microscope =……..N m^{-2}

Result: Young's modulus of the material of the given beam using telescope and optical lever =……. N m^{-2}

Which of these results is closer to the standard value? Theoretically, the accuracy to which the depression is measured using a microscope is equal to the least count (L.C.) of the microscope. Suppose that L.C. of microscope is 0.001 cm.

In the case of optical lever arrangement, the least count of vertical scale is 0.1 cm. This is multiplied by the factor *x*/2*D* (see Observation Table 3.2). If *D* = 1m = 100 cm and *x* = 3 cm, then $\frac{x}{2D} = \frac{3}{200} = 0.015$. So the least count of measurement of depression by the optical lever arrangement = 0.1 × 0.015 = 0.0015 cm.

This shows that measurement of depression with microscope (and hence value of *Y*) is more accurate than with an optical lever arrangement. But the optical lever method can be made to give better results than microscope method. For this you have to think of a way to improve the least count for the measurement of depression by optical lever arrangement. You may, for instance, use a half-millimetre scale instead of mm scale. The least count of the measurement of depression with the optical lever arrangement depends on (i) the length of tilting arm of the optical lever, *x*, and (ii) the distance between the mirror and the scale *D*. It may not be possible to adjust *x* unless you can use another optical lever. However, if we use a high power telescope so that *D* can be as large as possible, say 3 m, the optical-lever method can yield more accurate results.

EXPERIMENT 4

DETERMINATION OF THE MODULUS OF RIGIDITY OF A WIRE USING MAXWELL'S NEEDLE

Structure

4.1 Introduction
Expected Skills
4.2 Familiarization with Maxwell's Needle Apparatus
4.3 Theory of Modulus of Rigidity
4.4 Experiment with Maxwell's Needle Apparatus

4.1 INTRODUCTION

In the previous experiment, you have learnt how to determine Young's modulus of steel using the method of bending of beams. The depression produced in the beam loaded with weights at the centre could be measured either using a travelling microscope or an optical lever arrangement. You may recall that this information is vital in the construction of buildings as well as bridges. In this experiment you will determine the **modulus of rigidity** of a wire using Maxwell's needle apparatus. When you visit the physics laboratory at your study centre, look out for equipment wherein modulus of rigidity plays some role in the determination of a physical quantity. In particular, look out for torsional pendulum, which is used to determine modulus of rigidity of a wire and Searle's apparatus to determine elastic constants. You can also look for ballistic galvanometer, which is used to study weakly damped motion

Expected Skills

After performing this experiment, you should be able to:

- set up Maxwell's needle apparatus;
- configure it for different mass distributions;
- measure the diameter of wire with a screw gauge;
- use physical balance for measuring mass; and
- take time period readings for harmonic oscillations.

The apparatus required to measure modulus of rigidity using Maxwell's needle is listed below.

Apparatus required

Maxwell's needle, a thin long wire whose modulus of rigidity is to be determined, metre scale, stop watch, physical balance, weight box, screw gauge, knitting needle fitted vertically on a stand, telescope.

4.2 FAMILIARIZATION WITH MAXWELL'S NEEDLE APPARATUS

In this experiment Maxwell's needle is used to determine the modulus of rigidity of a wire. It essentially uses dynamical method where time periods of the needle are measured under different configurations.

Refer to Fig. 4.1. You will note that Maxwell's needle consists of a hollow brass tube of length *L*. It is suspended horizontally by a wire fixed to its centre. The other end of the wire is clamped to a rigid support. The tube carries four cylinders of identical length (= *L*/4). Two cylinders (*H*, *H*) are hollow of mass m_1, say. Other two cylinders are solid (*S*, *S*) and their mass $m_2 > m_1$. A plane mirror *M* is fixed at the middle of the needle on its top. Usually a knitting needle is placed vertically and its image can be seen in the mirror through a telescope mounted at adequate height/position. You will be required to use a stop watch to note down the time taken by the Maxwell's needle to complete certain number of oscillations and a screw gauge to measure the radius of the wire.

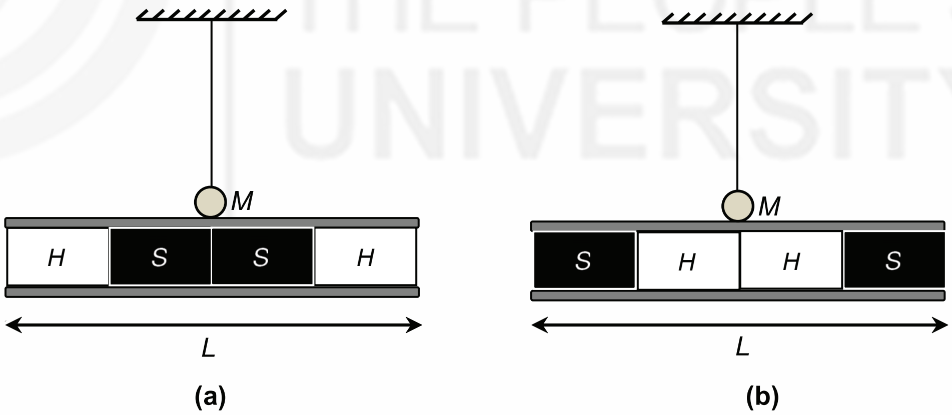


Fig. 4.1: Maxwell's needle apparatus

Before going to the procedure of measurements, let us briefly discuss the theory behind this experiment.

4.3 THEORY OF MODULUS OF RIGIDITY

We know that all bodies are deformed when a force is applied on them. Depending on the direction of application of force, we define different elastic constants. For instance, when the magnitude of applied force is within the

> You may recall from your +2 physics that
>
> $$\text{Elasticity} = \frac{\text{Stress}}{\text{Strain}}$$
>
> Stress is defined as restoring force developed within the body per unit area and strain is fractional change in length, volume or shape of a body. The corresponding strains are characterized by Young's modulus, bulk modulus and modulus of rigidity.

elastic limit, all bodies regain their original state on removal of deforming force due to the restoring force that develops in the body. Young's modulus, *Y*, characterises the effect of applied force in the form of change in length. It is defined as the ratio of stress to longitudinal strain.

When the applied force produces a change in the shape of a body, leaving its volume constant, the strain is characterised by the angle of shear. Then the ratio of stress to shearing strain defines the **modulus of rigidity** or **shear modulus.** We will use Maxwell's needle to measure the modulus of rigidity of a wire. For this purpose, we use it in different configurations by changing the position of its cylinders, thereby changing the mass distribution on needle.

The solid cylinders are placed in the inner positions and the hollow cylinders in the outer positions in the tube as shown in Fig. 4.1a. When the wire is twisted, the system begins to execute torsional oscillations about the wire as the axis of oscillation. The motion of the Maxwell's needle will be simple harmonic and the time period of oscillation is given by

$$T_1 = 2\pi\sqrt{I_1 / C} \tag{4.1}$$

where I_1 is moment of inertia of the suspended system and C is restoring couple per unit twist of the wire due to torsional reaction.

Next, the arrangement of solid and hollow cylinders is interchanged so that solid cylinders are on the outside and the hollow cylinders are in the interior (Fig. 4.1b). If the needle is made to oscillate again with changed configuration, we expect the time period to be different from the earlier one. Do you know that the moment of inertia will be different from the earlier case as mass distribution about the axis of rotation has changed? If we denote the moment of inertia in this case by I_2 and make Maxwell's needle to oscillate as before, the new time period will be given by

$$T_2 = 2\pi\sqrt{I_2 / C} \tag{4.2}$$

On squaring Eqs. (4.1) and (4.2) and combining the resulting expressions, we can write

$$T_2^2 - T_1^2 = \frac{4\pi^2}{C}(I_2 - I_1)$$

$$\Rightarrow \quad C = \frac{4\pi^2(I_2 - I_1)}{T_2^2 - T_1^2} \tag{4.3}$$

We can relate the restoring couple per unit twist with modulus of rigidity. To do so, we think of what happens when the wire is twisted in a plane perpendicular to its length. Due to elasticity, an equal and opposite torque develops in the wire. In the equilibrium position, the twisting couple is equal and opposite to the restoring couple. The restoring torque per unit radian is given by

$$C = \frac{\pi n r^4}{2\ell} \tag{4.4}$$

where n is the modulus of rigidity, r is the radius of the wire and ℓ is its length.

On combining Eqs. (4.3) and (4.4), we can write

$$n = \frac{8\pi\ell(I_2 - I_1)}{(T_2^2 - T_1^2)r^4} \tag{4.5}$$

From this result, you will note that to determine n, we must know I_1 and I_2. Normally, it is not easy to determine the moment of inertia of a body accurately and any error gets magnified because of the difference $(I_2 - I_1)$. However, in this method, we can counter this difficulty by expressing the difference $(I_2 - I_1)$ in terms of the difference of masses of the cylinders $(m_2 - m_1)$ and the length of the Maxwell needle tube. To understand this, note that the centres of mass of the inner and outer cylinders lie at distance $L/8$ and $3L/8$, respectively, from the axis of oscillation. Therefore, the essential change from the first configuration, where solid cylinders occupy the inner positions, to the second configuration, when they occupy the outer positions, consists of transferring mass $(m_2 - m_1)$ from a distance $L/8$ to a distance $3L/8$ from the axis of oscillation on either side of it. Due to this, the moment of inertia of the loaded tube changes. Using the principle of parallel axis (read the margin remark) in the instant case, we can write

Principle of parallel axes states that the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the distance between the two axes. You may have studied this principle in your Class 12.

$$I_2 = I_1 + 2(m_2 - m_1)\left[\left(\frac{3L}{8}\right)^2 - \left(\frac{L}{8}\right)^2\right]. \tag{4.6}$$

Note that we have multiplied the mass by a factor of 2 to account for the change taking place on both sides of the axis of rotation.

On simplification, we can rewrite Eq. (4.6) as

$$I_2 - I_1 = 2(m_2 - m_1)\times\left(\frac{9L^2}{64} - \frac{L^2}{64}\right)$$

$$= (m_2 - m_1)\frac{L^2}{4} \tag{4.7}$$

On combining Eqs. (4.5) and (4.7), we get

$$n = \frac{2\pi\ell(m_2 - m_1)L^2}{(T_2^2 - T_1^2)r^4} \tag{4.8}$$

Eq. (4.8) expresses modulus of rigidity in terms of $\ell, m_1, m_2, L, T_1, T_2$ and r. These physical quantities can be determined accurately and therefore Maxwell's needle provides us a fairly precise method for determination of modulus of rigidity. Note that

1. The physical quantities occur with different powers. For instance, radius of the wire occurs in the fourth power and a small error in its determination will affect the result significantly. Therefore, you must determine its value very carefully and we recommend the use of a screw gauge for this purpose.

2. In the formula of modulus of rigidity, difference of the squares of time periods occurs in the denominator. Since it will be very small, it is important to obtain values of T_1 and T_2 by noting time for 20 oscillations.

4.4 EXPERIMENT WITH MAXWELL'S NEEDLE APPARATUS

To measure the modulus of rigidity using Maxwell's needle, you should perform the following steps:

1. Take a long kink-free wire (about 1 m) of the material whose modulus of rigidity is to be determined. (If there is any kink, it must be removed before you begin your experiment. A simple method could be to hold the wire in the folds of a handkerchief and pull it along the length. If you succeed, fine. Otherwise, request your Counsellor to get it changed.)

2. Suspend the wire from a rigid support and fasten the mid point of hollow tube of Maxwell's needle to its other end. You must ensure that the needle is horizontal and the mirror faces you.

3. Place a knitting needle in front of the mirror so that its tip is in the middle of the mirror. (It will act as an indicator while counting the number of oscillations.) Now focus a telescope from a distance of about 1.5 m on the image of the tip of the needle formed in the mirror and remove parallax, if any.

4. Using the weighing balance measure the masses of two hollow and two solid cylinders separately. Note the masses of hollow cylinders under m_1 and that of solid cylinders under m_2 in Observation Table 4.1. Calculate average masses of m_1 and m_2.

5. Put the solid cylinders *SS* in the middle and the hollow cylinders *HH* on the outer side. You must ensure that

   i) the Maxwell's needle is horizontal, and

   ii) no part of the cylinders projects outside the tube.

   (If the needle is not horizontal, up and down motion of the needle will lead to erroneous results.)

6. Push one end of Maxwell's needle slightly backward in a horizontal plane and let it go. The needle will begin to execute torsional oscillations.

7. When the motion becomes steady, i.e. there is neither up and down nor lateral motion, determine the time for 20 oscillations (t_1). To do so, you should start the stop watch when the image of the needle crosses towards left or right vertical cross-wire of the telescope and count the time taken for 20 oscillations. (You must have learnt to use a stop watch in your +2 classes.) Record your reading in the Observation Table 4.1A. You should switch off the fan while taking readings.

8. Repeat steps 6 and 7 at least five times.

9. Next repeat steps 6 to 8 above by interchanging the position of hollow and solid cylinders and noting the readings for 20 oscillations under t_2.

10. Measure the length of Maxwell's tube (L) as well as the length of the wire (ℓ) and record these in Observation Table 4.1.

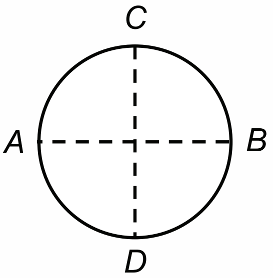


Fig. 4.2: Cross section of a wire

11. Since the fourth power of the radius of the wire occurs in the formula of rigidity modulus and its value is very small, you must measure its diameter very accurately. Take readings for at least five different positions along the length of the wire using a screw gauge. For greater accuracy, measure the diameter at each position in two mutually perpendicular directions, *AB* and *CD* as shown in Fig. 4.2. This will help you to minimise the effect of non-uniformity in the cross-section of the wire.

12. Calculate the modulus of rigidity using Eq. (4.8).

Observation Table 4.1

Length of the wire, ℓ = ……………………… cm

Length of Maxwell's tube, L = ……………………… cm

Average mass of hollow cylinders, $m_1 = \dfrac{.....+.....}{2} = ...$g

Average mass of solid cylinders, $m_2 = \dfrac{.....+.....}{2} = ...$g

A. Determination of Time Periods

No. of oscillations, n = 20

Least count of stop watch = …….. s

Sl.No.	**Time t_1 for n oscillations (s)**	**Time t_2 for n oscillations (s)**	$\frac{t_1}{n} = T_1$**(s)**	$\frac{t_2}{n} = T_2$**(s)**
1.				
2.				
3.				
4.				
5.				

Average value of T_1 = ………… s

Average value of T_2 = ………… s

B. Determination of Radius of Wire

Least count of screw gauge = ………….. cm

Sl.No.	Diameter (cm)		Radius (cm)	
	Along *AB*	Along *CD*	Along *AB*	Along *AB*
1.				
2.				
3.				
4.				
5.				

Average radius along *AB*, r_1 = …………….. cm

Average radius along *CD*, r_2 = …………….. cm

Mean value $r = \frac{r_1 + r_2}{2}$ cm

Calculations:

Modulus of rigidity of wire (from Eq. 4.8)

$$n = \frac{2\pi\ell(m_2 - m_1)L^2}{(T_2^2 - T_1^2)r^4}$$

Result: The modulus of rigidity of wire = ………… $\times 10^{11}$ dyne cm^{-2}.

EXPERIMENT 5

DETERMINATION OF ELASTIC CONSTANTS OF A WIRE BY SEARLE'S METHOD

Structure

5.1 Introduction
Expected Skills
5.2 Familiarisation with Searle's Apparatus
5.3 Theory of Elastic Constants
5.4 Measurements with Searle's Apparatus
5.5 Calculations

5.1 INTRODUCTION

In earlier experiments, you have learnt how to determine Young's modulus (Y) and modulus of rigidity (n) of a material using the method of bending of beams and Maxwell's needle, respectively. There are two other elastic constants: Bulk modulus (K) and Poisson ratio (σ). You may now ask: Can we determine all these elastic constants in one experiment? The answer to this question is yes, we can. In a physics laboratory, Searle's apparatus is used to determine Y, n, σ and K. In this experiment you will learn to use it for obtaining various elastic constants.

> Within elastic limits, **bulk modulus** is defined as the ratio of stress to volume strain:
>
> $$K = \frac{\text{Stress}}{\text{Vol. Strain}} = \frac{F / A}{(\Delta V / V)}$$
>
> **Poisson ratio** is defined as the ratio of lateral strain to linear or tangential strain. It is denoted by the symbol σ. Its value lies between 0.2 and 0.4 and it has no units.

Expected Skills

After performing this experiment, you should be able to:

- list the constituents of Searle's apparatus;
- assemble Searle's apparatus;
- set up the constraint system and bring it in oscillation mode and obtain the value of Young's modulus;
- set up the apparatus to take measurements for determining modulus of rigidity;
- determine Poisson ratio from the values of Young's modulus and modulus of rigidity; and
- calculate the bulk modulus from obtained values of Young's modulus and Poisson ratio.

We now list the apparatus required to perform this experiment.

Apparatus required

Two identical rods (bars) of circular (or square) cross-section with arrangement to hang, a thin wire of about 30 cm length, a stop watch, a screw gauge, a vernier callipers, inextensible thread, metre scale, weight box and a physical balance.

5.2 FAMILIARIZATION WITH SEARLE'S APPARATUS

Refer to Fig. 5.1a. It shows two identical rods *AB* and *CD* joined at their centres by a wire *FH* (of length *L*). We have to determine elastic constants of the material of this wire. This system is suspended horizontally from a rigid support by two parallel torsionless vertical inextensible threads, EF' and GH', preferably of silk, so that when the wire is straight and the system is in equilibrium position, the rods will be parallel to one-another in the plane *ABDC*. The threads are attached to small needle holes at F' and H' at the middle of the rods. The rods are turned through a small equal angle (θ) in opposite directions in a horizontal plane. For this, ends *A* and *C* are drawn towards each other, as shown in Fig. 5.1b.

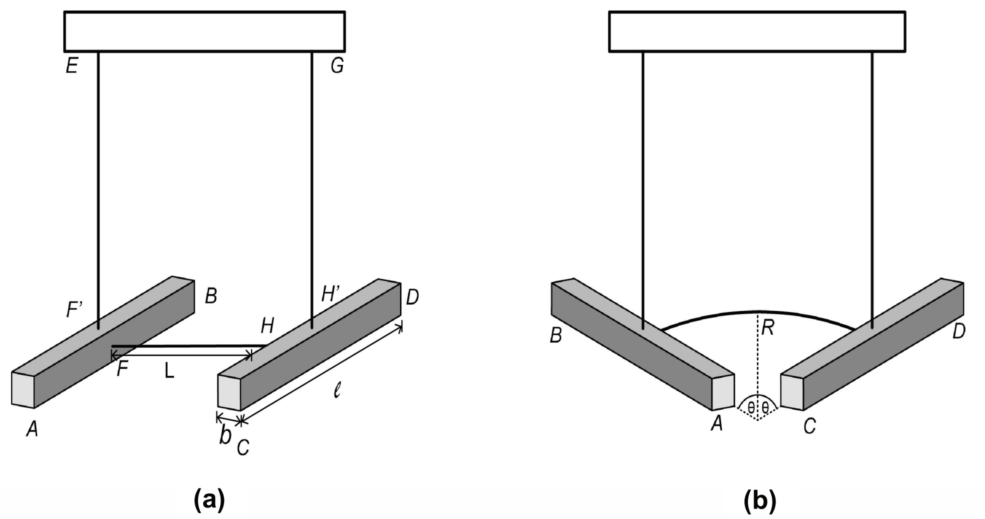


Fig. 5.1: Searle's apparatus for determination of elastic constants: a) equilibrium position; and b) instantaneous constrained configuration.

In this constrained condition, the wire *FH* bends into a circular arc. As *A* and *C* are released, the rods begin to execute torsional oscillations. By taking the reading of time period of these oscillations, we can arrive at the elastic constants of the wire. Now we will discuss briefly the theory governing these measurements.

5.3 THEORY OF ELASTIC CONSTANTS

When we set the rods in torsional oscillations, the middle points of bars remain essentially at rest. However, if the amplitude of oscillations is small, the arc will almost resemble a straight line. This means that the distance between the lower ends *F* and *H* of the supporting threads remains practically constant implying that the threads in Fig. 5.1b remain vertical during the oscillations of the rods/bars. This suggests that under small oscillation approximation, no horizontal components of tensions in the threads act on the wire.

Note that the mass of the wire is negligible compared with that of the rods and the mid-points of the rods remain at rest, i.e., there is no horizontal or vertical motion. Therefore, the action of the wire on either rod manifests as a couple. The moment of this couple is the same at every point of the wire and it bends into a circular arc.

If the radius of the arc is *R* and the angle of deflection of each rod from its respective equilibrium position is θ, we can write

$$L = R\,2\theta \tag{5.1a}$$

where *L* is length of the wire and 2θ denotes the angle subtended by the wire at the centre of curvature of the circular arc into which it bends.

By rearranging Eq. (5.1a), we can write

$$R = \frac{L}{2\theta} \tag{5.1b}$$

In the experiment of bending of beam (Sec. 3.2.2), we have already obtained the expression for the bending moment of a beam (Eq. 3.6). Applying the same arguments, the bending moment of the wire and the couple exerted by it on each rod is given by:

$$G = \frac{Y I_g}{R} = Y\frac{\pi r^4}{4R} = \frac{Y\pi r^4\theta}{2L} \tag{5.2}$$

where *Y* is Young's modulus for the material of the wire and $I_g (= \pi r^4/4)$ is the geometrical moment of inertia of the area of cross-section of the wire about an axis passing through the centre of the area and normal to the plane of bending.

This couple produces an angular acceleration $\frac{d^2\theta}{dt^2}$ in each rod directed towards its equilibrium position. Hence, we can write

$$I\frac{d^2\theta}{dt^2} = -\frac{Y\pi r^4\theta}{2L} \tag{5.3}$$

where I is the moment of inertia of each rod about an axis passing through its centre of gravity (C.G.) and perpendicular to its length, ℓ, i.e., about the thread from which it is suspended. We can rewrite Eq. (5.3) as

$$\frac{d^2\theta}{dt^2} = -\frac{Y\pi r^4}{2IL}\theta \qquad (5.4)$$

Do you recognise this equation? It represents S.H.M and can be written as

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \text{ where } \omega^2 = \frac{Y\pi r^4}{2IL}.$$

The period of harmonic oscillation of each rod is given by

$$T_1 = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2IL}{Y\pi r^4}}$$

$$\Rightarrow \quad Y = \frac{8\pi L I}{T_1^2 r^4} \qquad (5.5)$$

If the rods have square or rectangular cross-section with length ℓ and breadth b, we can write

$$I = M\left[\frac{\ell^2 + b^2}{12}\right] \qquad (5.6a)$$

where M is mass of the rod.

If the cross-section of the rods is circular with radius R_c, we can write

$$I = M\left(\frac{\ell^2}{12} + \frac{R_c^2}{4}\right) \qquad (5.6b)$$

Note that Eq. (5.5) provides us a method of determining **Young's modulus** *Y* of the material of the wire *FH*. You may now ask: How can we determine other elastic constants using this apparatus? To obtain the modulus of rigidity, remove the rods from their suspensions and fix one of these horizontally to a rigid support, as shown in Fig. 5.2. The wire is hung vertically with the other rod suspended horizontally at its lower end.

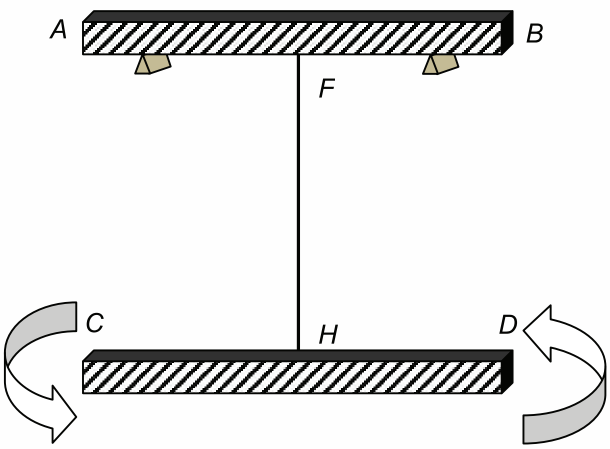


Fig. 5.2: Arrangement of Searle's apparatus for obtaining modulus of rigidity.

Now suppose the wire is twisted through a small angle by moving the rod *CD* in a plane perpendicular to its length in anticlockwise direction. The rod will begin to execute oscillations in the horizontal plane on being released. Do you recognise this system and say how it will act? This arrangement works like a torsional pendulum.

For SHM, the time period of oscillation is given by

$$T_2 = 2\pi\sqrt{\frac{I}{C}} \tag{5.7}$$

where I is moment of inertia of rod *CD* about the axis of rotation passing through its centre and $C = \frac{\pi n r^4}{2L}$ is the restoring torque per unit radian in the wire of radius *r*, length *L* and modulus of rigidity *n*. As explained in Experiment 4, the restoring torque develops in the wire due to elasticity. On squaring the expression given in Eq. (5.7), we can write

$$C = \frac{\pi n r^4}{2L} = \frac{4\pi^2 I}{T_2^2}$$

so that $$n = \frac{8\pi I L}{T_2^2 r^4} \tag{5.8}$$

On combining Eqs. (5.5) and (5.8), we get

$$\frac{Y}{n} = \frac{T_2^2}{T_1^2} \tag{5.9}$$

The ratio $\frac{Y}{n}$ is known to be connected to Poisson ratio, σ, through the relation

$$\frac{Y}{n} = 2(1+\sigma) \tag{5.10}$$

so that $$\sigma = \frac{Y}{2n} - 1 = \frac{T_2^2}{2T_1^2} - 1 \tag{5.11}$$

Also the bulk modulus, *K* and Young's modulus, *Y* are related to Poisson ratio as

$$\frac{Y}{3K} = 1 - 2\sigma \tag{5.12}$$

so that $$K = \frac{1}{3}\frac{Y}{(1-2\sigma)} \tag{5.13}$$

It is important to note that *Y*, *n* and σ can be determined using Searle's apparatus by simply observing the time periods of oscillation in two different configurations. And the bulk modulus can be obtained from the measured values of *Y* and σ. To that extent, this method is rather straightforward.

5.4 MEASUREMENTS WITH SEARLE'S APPARATUS

1. Set up the apparatus as shown in Fig. 5.1a so that the rods *AB* and *CD* lie in the same horizontal plane. For this, you can take two inextensible cotton or silk threads of about 60 cm length.

2. Set up a pointer close to the centre of one of the rods and make a mark on the rod in line with the pointer when the Searle's apparatus is in equilibrium position and the rods are at rest.

3. Pass a cotton loop around the ends *A* and *C*. This will help you to draw these ends towards each other through a small angle. Make sure that the constrained system is at rest.

4. Burn the cotton loop. The ends *A* and *C* will become free and the rods will begin to oscillate. Make sure that the amplitudes of oscillations of these rods are small, about 3″. Note the least count of the stop watch and use it to observe time (t) for say 20 oscillations and record it in Observation Table 5.1.

5. Repeat Steps (3) and (4) outlined above at least five times. From these you can calculate the mean value of time period $T_1 = \dfrac{t_1 + t_2 + t_3 + t_4 + t_5}{5 \times 20}$.

6. Remove the (cotton or silk) threads supporting the rods in the suspension configuration and clamp one of the rods, say *AB*, horizontally with a rigid support, as shown in Fig. 5.2. You must make sure that the wire is vertical.

7. Twist the wire by rotating one end of the rod *CD* in a horizontal plane through a small angle and then release it. The rod will execute torsional oscillations. Before you start noting time (t') for say, 20 oscillations, you must make sure that the rod does not wobble.

8. Repeat Step 7 at least five times and record your readings in Observation Table 5.1.

9. Measure the diameter of the wire at a minimum of five different places along the length of the wire in two mutually perpendicular directions *XX′* and *YY′* using a screw gauge. This will help in minimising the effect of inhomogeneity in wire thickness. Record your readings in Observation Table 5.2.Note that a small error in the radius will significantly influence the values of elastic constants since its fourth power occurs in their expressions. Moreover, its magnitude is small; therefore, you must determine r very carefully.

   Measure the length of the wire under study (L) using a metre rod and note it under Observation Table 5.2.

10. Determine mass M and length ℓ of the rod *CD* accurately and record these at the top of the Observation Table 5.3. To measure the radius R_c

of rod *CD*, you can use vernier callipers. Take at least five readings at different places and record these in Observation Table 5.3.

Observation Table 5.1: Measurement of time periods T_1 and T_2

No. of oscillations = 20
Least count of stop watch = …….. s

Sl.No.	t	t'
1.		
2.		
3.		
4.		
5.		

$T_1 = \frac{\Sigma t}{100} =$ ....s $\qquad$ $T_2 = \frac{\Sigma t'}{100} =$ ....s

Observation Table 5.2: Measurement of radius of wire

Least count of screw gauge = ………….. cm

Sl.No.	Diameter (cm)		Mean diameter (cm)
	Along XX'	Along YY'	$d = \frac{XX' + YY'}{2}$
1.			d_1
2.			d_2
3.			d_3
4.			d_4
5.			d_5

Mean diameter of wire = d = …………….. cm

Mean radius of wire = $r = \frac{d}{2}$ = …………….. cm

Length of wire = L = …………….. cm

Observation Table 5.3: Measurement of radius of rod

Mass of rod $CD = M =$ …………….. g

Length of the rod CD, $\ell =$ …………….. cm

Least count of vernier callipers = ……………. cm

Sl.No.	Diameter (cm)		Mean diameter (cm)
	Along XX'	Along YY'	
1.			D_1
2.			D_2
3.			D_3
4.			D_4
5.			D_5

In case, the rod has rectangular cross section, you will measure its breadth using vernier callipers and use Eq. (5.6a) to calculate I.

Mean diameter of the rod CD = D = …………….. cm

Mean radius of rod $CD = R_c = \frac{D}{2} =$ …..…………….cm

5.5 CALCULATIONS

Use the following formulae for calculating the values of various elastic constants:

i) $$Y = \frac{8\pi L}{r^4 T_1^2} M\left(\frac{\ell^2}{12} + \frac{R_c^2}{4}\right) \text{ dyne cm}^{-2}$$

ii) $$n = \frac{8\pi L}{r^4 T_2^2} M\left(\frac{\ell^2}{12} + \frac{R_c^2}{4}\right) \text{ dyne cm}^{-2}$$

iii) $$\sigma = \frac{Y}{2n} - 1$$

iv) $$K = \frac{Y}{3(1-2\sigma)} \text{ dyne cm}^{-2}.$$

Result: The values of elastic constants determined using Searle's apparatus are as follows:

$Y =$ .... dyne cm^{-2}

$n =$ .... dyne cm^{-2}

$\sigma =$ ....

$K =$ .... dyne cm^{-2}

On knowing these values you can determine the material of the wire. For this you should look at standard values given in a laboratory manual or consult your Counsellor.

EXPERIMENT 6

DETERMINATION OF ACCELERATION DUE TO GRAVITY USING BAR PENDULUM

Structure

6.1 Introduction

Expected Skills

6.2 Theory of Compound Pendulum

6.3 Procedure for Determining Gravitational Acceleration

Bar Pendulum

Measuring Oscillations

Setting up and Measurements with Bar Pendulum

Determination of the Radius of Gyration

6.1 INTRODUCTION

In Experiment 5, you have learnt how to determine the elastic constants (Young's modulus, Modulus of rigidity, Bulk modulus and Poisson ratio) using Searle's apparatus. All these elastic constants were determined in one experiment. Now, in this experiment, you will work with oscillating systems.

You might recall that in your school physics course you studied only the dependence of period on the length of simple pendulum.

In your school, you must have worked with a simple pendulum. You know that a simple pendulum is a heavy (point) mass suspended from a rigid support by a weightless, inextensible string. In practice, a simple pendulum is made up of heavy metallic bob suspended from a rigid support by means of an ordinary string. (So you must appreciate that what we have in practice is not an ideal simple pendulum!) It can freely oscillate to and fro about the point of suspension in a plane. A pendulum, as you know, happens to be the main equipment inside a wall clock. The maximum displacement of the bob on either side of its equilibrium position is called the **amplitude** of oscillation. The time taken by the pendulum to complete one oscillation is called **time period**.

You may think that a simple pendulum is an ideal arrangement for time measurement. But it is not so; a practical simple pendulum has some inherent drawbacks, which we discuss in Sec. 6.2. Use of a compound pendulum eliminates some of these drawbacks. In this experiment, we shall restrict ourselves to the oscillatory motion of a bar pendulum. Using the bar pendulum, you will measure acceleration due to gravity as described in Sec. 6.3. In the next experiment, you will determine acceleration due to gravity using a Kater's pendulum which has inhomogeneous mass distribution.

Expected Skills

After performing this experiment, you should be able to:

- establish the relation between the time period and the length of a bar pendulum;
- discover the dependence of the period on the length of a bar pendulum;
- compute the value of acceleration due to gravity using a bar pendulum; and
- compute the radius of gyration of a bar pendulum.

The apparatus you will require for this experiment is listed below.

Apparatus required

Bar pendulum, stop watch, metre scale.

6.2 THEORY OF COMPOUND PENDULUM

We know that a simple pendulum suffers from the drawback that some air is always dragged by the bob. Similarly, the string may not be perfectly inextensible leading to non-planar oscillations and motion about the point of suspension may have rotational component, etc. These sources of error sometimes lead to variation in the value of *T*. Can you suggest a way to overcome these problems? The remedy lies in the use of a compound pendulum.

Consider a compound pendulum of mass *m* suspended from point *S*. Suppose it is given a small angular displacement θ as shown in Fig. 6.1. Now the centre of gravity (CG) G, is shifted to point *G′* with gravitational force *mg* acting in downward direction.

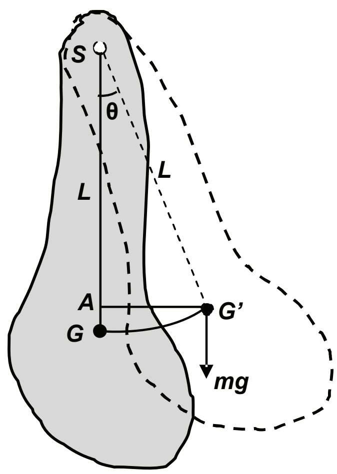


Fig. 6.1: Compound pendulum in motion.

When the distance *SG* = *SG′* = *L*, the torque experienced is

$$\tau = -mg \times AG'$$

$$= -mgL \sin\theta$$

For small angle θ, we can approximate $\sin\theta \approx \theta$ and hence

$$\tau = -mgL\theta \qquad \text{(i)}$$

If the angular acceleration due to torque is α and the moment of inertia of the rigid body about S is I, we can write

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} \qquad \text{(ii)}$$

Hence we have,

$$I\frac{d^2\theta}{dt^2} = -mgL\theta$$

or

$$\frac{d^2\theta}{dt^2} = -\frac{mgL}{I}\theta \qquad \text{(iii)}$$

This equation represents a simple harmonic motion, and can be expressed in terms of oscillation frequency ω_0 as given by

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0 \qquad (6.1)$$

where $\omega_0^2 = \dfrac{mgL}{I}$

Now the period of oscillation T is related to ω_0 by the relation

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{mgL}} \qquad (6.2)$$

To obtain T, we should express the moment of inertia I in terms of measurable quantities. You know that the moment of inertia of a body about a given axis (I) and its moment of inertia about the axis passing through its CG (I_g) are related by the following relation:

$$I = I_g + mL^2 = m(k_r^2 + L^2) \qquad (6.3)$$

where L is the distance between the two axes, $I_g = mk_r^2$ and k_r is the radius of gyration of the body about an axis passing through G.

The radius of gyration is the radial distance between the axis and the point at which the whole mass of the body could be placed without any change in the moment of inertia of the body about that axis.

Substituting for I from Eq. (6.3) in Eq. (6.2) we get

$$T = 2\pi\sqrt{\frac{m(k_r^2 + L^2)}{mgL}} = 2\pi\sqrt{\frac{k_r^2 + L^2}{gL}}$$

$$= 2\pi\sqrt{\frac{\frac{k_r^2}{L} + L}{g}} \qquad (6.4)$$

Substituting $\frac{k_r^2}{L} + L = L'$, we get

$$T = 2\pi\sqrt{\frac{L'}{g}} \tag{6.5}$$

You will recall that this is the expression for the period of oscillations of a simple pendulum. Length L' is termed as **the equivalent length of a simple pendulum** for the given compound pendulum. This means that the time period of a simple pendulum of length $L'\left(= \frac{k_r^2}{L} + L\right)$ is the same as that of the compound pendulum with radius of gyration k_r and with distance L between the point of suspension and the centre of gravity.

When the distance between the point of suspension S and centre of gravity G is L, a point P exists on the other side of G at the distance $\frac{k_r^2}{L}$, which corresponds to same time period of oscillation. It is the **centre of oscillation**. The distance between S and P is

$$= L + \frac{k_r^2}{L} (= L')$$

Hence the centre of oscillation lies at the equivalent length of the simple pendulum (L') from the point of suspension. Since the period of oscillation is same for both these points, we can use them interchangeably.

Therefore, when the compound pendulum is made to oscillate about a horizontal axis, its motion is simple harmonic and the time period is given by

$$T = 2\pi\sqrt{\frac{k_r^2 + L^2}{Lg}} \tag{6.6}$$

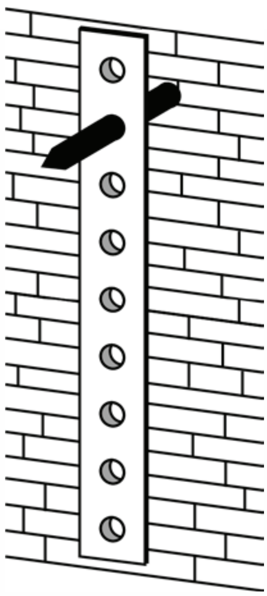
where L is the distance between the point of suspension and CG and k_r is the radius of gyration of the body.

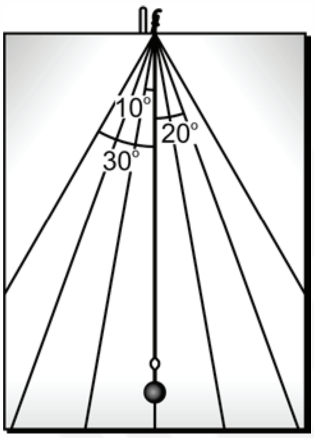
Eq. (6.6) is a general equation for the time period of a compound pendulum. Now we define

$$L' = \frac{k_r^2}{L} + L$$

and call it the **length of an equivalent simple pendulum**. The time period is given by

$$T = 2\pi\sqrt{\frac{L'}{g}} \tag{6.7}$$


(a)



(b)

Fig. 6.2: a) A bar pendulum; b) pendulum with angles graduated sheet.

In this part of the experiment, you are required to investigate how the period of oscillation of a bar pendulum varies with distance between its point of suspension and CG.

6.3 PROCEDURE FOR DETERMINING GRAVITATIONAL ACCELERATION

6.3.1 Bar Pendulum

A bar pendulum is a rigid body capable of oscillating freely about a horizontal axis. In the physics laboratory, it is normally available in the form of a bar of length nearly one metre and width about 2.5 cm. A series of circular holes, 5-6 mm in radius, are drilled symmetrically about its centre of gravity (CG), i.e. along the length of the bar as shown in Fig. 6.2a.) The centres of any two consecutive holes are at equal distances of about 5 cm. These holes allow the bar to be suspended from a knife-edge. Usually, two movable knife-edges are provided with the bar pendulum. These can be fitted successively in various holes, one on each side of CG and at equal distances from it. You may now realise how deficiencies in a simple pendulum are taken care of in a compound bar pendulum.

6.3.2 Measuring Oscillations

Make a reference mark using a pointer at the equilibrium position of the bar pendulum as well as at the maximum displacement of oscillation. You should keep the amplitude constant in each observation and it should be such that at no time, the small angle approximation is violated ($\theta \leq 10°$). That is, the motion should be simple harmonic. This may be ensured by using a protractor. (If a protractor is not available in the laboratory, you can make angle markings on a separate sheet of paper. Place the graduated scale behind the pendulum in such a way that the zero angle line coincides with the equilibrium position of the pendulum. Moreover, the origin of angular scale should be aligned with the point of suspension, as shown in Fig. 6.2b)

If you are working with another student, one of you can count while the other keeps time. The ‘counter’ should begin countdown two, one, “go”, one, two… and so on. This gives the timekeeper a warning about the ‘Go” signal. The end of counting may be indicated by saying ‘stop’. Make sure that each one of you takes at least one complete observation individually.

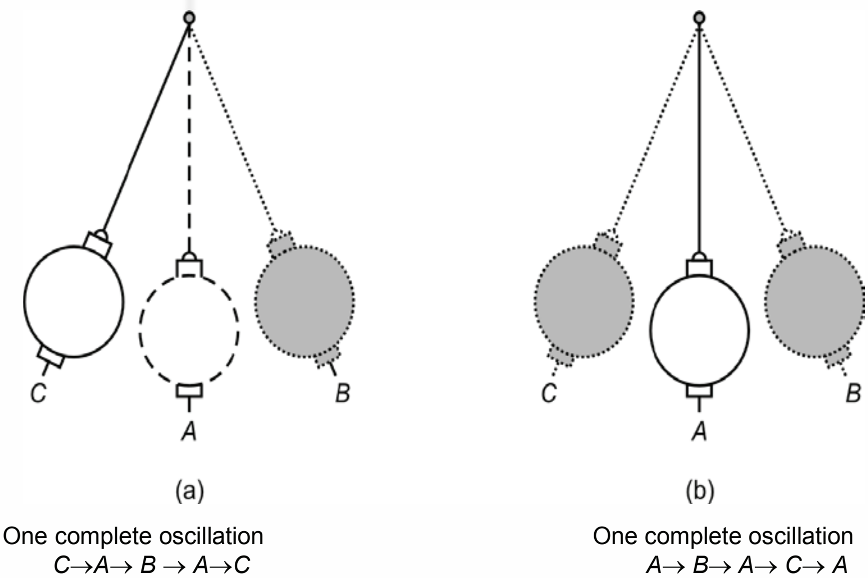


Fig. 6.3: Two different ways of counting the number of oscillations

To begin with, note the least count of the stop watch and record it in Observation Table 6.1. Now set the bar pendulum in motion by displacing it on one side. To count the number of oscillations, you can choose your reference point in two ways, as shown in Fig. 6.3. We prefer the second option because the reference point remains unaltered in this case.

The *reaction time* is the time interval between the input stimulus and its response.

Begin your counting through the *equilibrium position* of the bar pendulum. It is important to simultaneously start the stopwatch. (There can be time lag between the starting/stopping the watch and the oscillation count due to *reaction time*, which is, on an average, 0.3s. This can introduce some error in the value of time period *T*.) An important point to consider here is to know the degree of accuracy that is necessary. Another point is to measure a time interval in which the amplitude of swing does not diminish significantly. To see this, you can note time for 1, 10, 20, 30, 50, 70, 100 oscillations. You should take at least three observations in each case. Record your readings in Observation Table 6.1. Calculate the period of oscillation, *T*.

The least count of an ordinary stop-watch is of 0.1 s. So whenever you have to measure time of the order of one second or so, you should use a more accurate automatic switching device, such as digital timer.

Observation Table 6.1: Determination of optimum number of oscillations

Least count of stop watch = ……………………s

Sl. No.	No. of oscillations (*N*)	Time (s)				$T = \dfrac{\text{Mean Time}}{N}$ (s)
		(i)	(ii)	(iii)	(mean)	
1.	1					
2.	10					
3.	20					
4.	30					
5.	50					
6.	70					
7.	100					

Conclusion: The optimum number of oscillations is……………

To decide on the optimum number of oscillations, observe the variation in the value of *T*. When the difference between two successive values of *T* is less than 0.1 percent, it is acceptable. We expect the optimum number of oscillations to be 50. However, do not consider the number '50' to be sacrosanct. Make your own decision.

6.3.3 Setting up and Measurements with Bar Pendulum

To set up the bar pendulum and take measurements follow the steps given below.

1. Fix one knife-edge in the hole nearest to one end of the bar pendulum. The other knife-edge is fixed in the hole nearest to the other end so that the two knife-edges are equidistant from and symmetrically placed with respect to the CG of the bar.

2. Now suspend the pendulum vertically by resting it on one of the knife-edges on a horizontal rigid support.

3. As before, put a reference mark to denote the mean position of the pendulum.

4. Measure the distance between the point of suspension (centre of the hole) and the CG of the bar. This gives us *L*.

5. Displace the bar slightly aside and let it oscillate. You should ensure free oscillations in the vertical plane. Now you are ready to perform the experiment.

6. Now set the bar pendulum in oscillations by gently pushing the free end of pendulum from equilibrium position. Make sure that the pendulum oscillates in a plane parallel to the support wall of the pendulum and does not touch the wall.

7. Now measure the time for *N* (= 30) complete oscillations. Record your readings in Observation Table 6.2(a). Repeat this step three times.

8. Invert the pendulum and note the time for the same number of oscillations. Note the readings in Observation Table 6.2(b), which you will prepare on the lines of Observation Table 6.2(a).

9. Now insert the knife-edges in the adjacent holes so that they are symmetrical about CG, as before. You will note that now the length of the pendulum has been changed and the time of *N* oscillations is expected to be different from the preceding value.

10. Repeat observations by inserting the knife-edges in different holes and taking readings on either side of CG. At all times, the knife-edges should be symmetrical about CG. What happens as you approach the centre of the bar? You will observe that the time for *N* oscillations first decreases, takes a minimum value and then begins to increase. As you near the CG of the bar, it becomes very large.

11. See what happens when the knife-edge in put at the central hole. You will note that the bar will not oscillate; it just gets struck up on one side.

One complete oscillation of a pendulum is defined as the movement of pendulum from its equilibrium position to its extreme left and reaches to the extreme right and then comes to its equilibrium position again.

Observation Table 6.2(a): Variation of time period with distance of a hole from CG

Least count of the stop watch =…………..……….s

Sl. No.	Distance of the point of suspension from CG *L* (cm)	Time for oscillations *N* = 30				Time period *T* (s)	LT^2 (cm s^2)
		(i)	(ii)	(iii)	Mean		
1.							
2.							
3.							
4.							
5.							
6.							

Plot a graph between *T* and *L* on either side of CG as abscissae. You will get two curves which are symmetrical about the CG of the bar (Fig. 6.4). Now you draw a line parallel to the *x*-axis. At how many points does it intersect these curves? The number of points should be four, say at *J*, *K*, *M* and *N*, as shown in Fig. 6.4. At all these points, the period of the pendulum is the same.

Prepare a similar Observation Table 6.2(b) for noting the readings, taken by inverting the point of oscillation at each step.

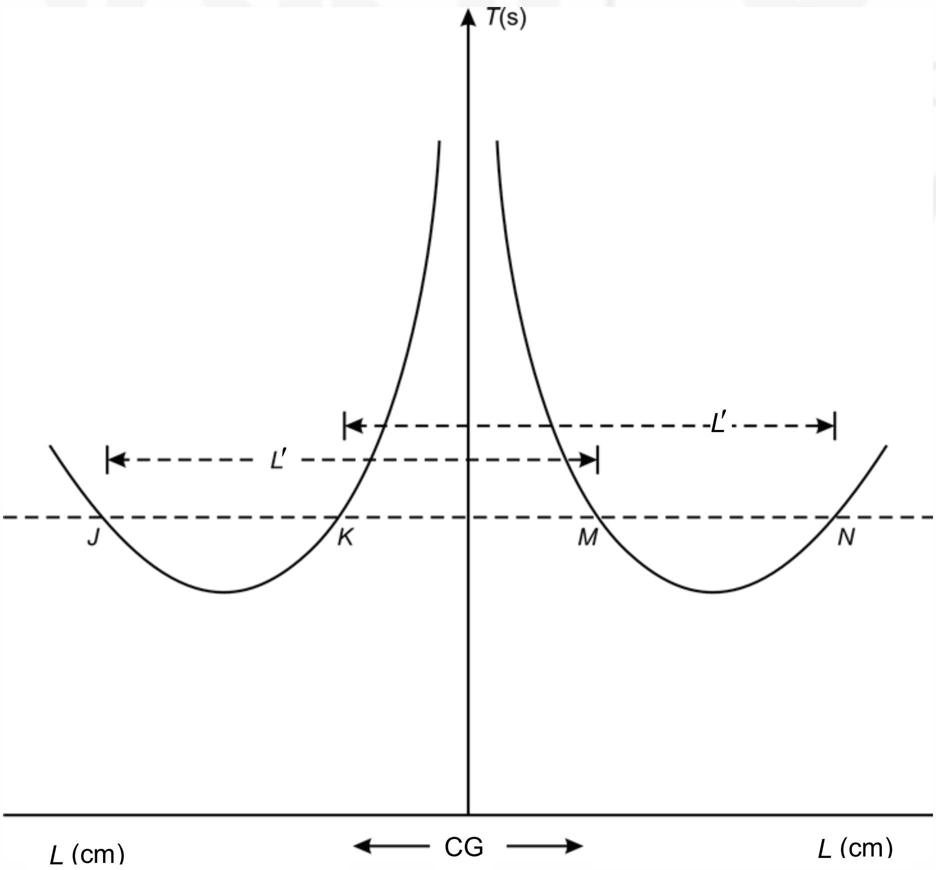


Fig. 6.4: Plot of time period with distance of point of suspension from CG.

Measure distances *JM* and *KN*. How do you interpret these? Each of these distances represents the length of an equivalent simple pendulum, L'. Using Eq. (6.7), you can compute the acceleration due to gravity.

Result: Acceleration due to gravity = ……………………………………......

6.3.4 Determination of the Radius of Gyration

To calculate the radius of gyration, we rewrite Eq. (6.6) as

$$LT^2 = \left(\frac{4\pi^2}{g}\right)L^2 + \frac{4\pi^2}{g}k_r^2 \qquad (6.8)$$

This equation suggests that if you plot LT^2 versus L^2, you will obtain a straight line, whose slope is $\frac{4\pi^2}{g}$ and intercept is $\frac{4\pi^2}{g}k_r^2 = c,$ say.

Hence

$$g = \frac{4\pi^2}{\text{slope}} \qquad (6.9)$$

and the radius of gyration is given by

$$k_r^2 = \frac{cg}{4\pi^2}$$

or

$$k_r = \frac{\sqrt{cg}}{2\pi} \qquad (6.10)$$

Result: i) The radius of gyration of the bar pendulum is ……... m

ii) The acceleration due to gravity is ……………………...ms^{-2}

SAQ 1 – **Bar pendulum**

i) Why is it necessary to put the knife-edges symmetrically about *CG*?

ii) Name two sources of error in your experiment.

EXPERIMENT 7

DETERMINATION OF ACCELERATION DUE TO GRAVITY BY KATER'S PENDULUM

Structure

7.1 Introduction
Expected Skills
7.2 Basic Theory and Construction of Kater's Pendulum
7.3 Experiment with Kater's Pendulum
Setting up the Apparatus
Taking Measurements

7.1 INTRODUCTION

You know that a rigid body performing harmonic oscillations about a point of suspension is known as compound pendulum. In the last experiment you worked with a bar pendulum, which is a regular shaped rigid body with its centre of gravity (CG) coinciding with its centroid (body centre). But in practice, the shape of the rigid pendulum can be arbitrary and the CG need not be situated at its centroid. One such rigid pendulum is shown in Fig. 7.1. When the pendulum is hanging freely, its CG shown by point *G* lies vertically below the point of suspension *S*. When this pendulum is given a *small* angular displacement, it performs simple harmonic motion about its equilibrium position.

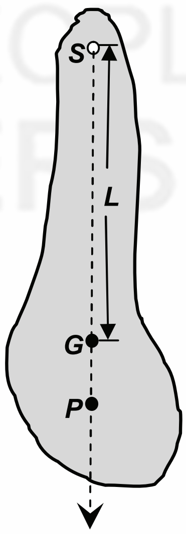


Fig. 7.1: Compound pendulum.

In such a pendulum, there exists another point *P* on the other side of CG, which has the same time period of oscillations. It is called the **centre of oscillation** (read the margin remark on the next page).

In the experiment on bar pendulum, you have learnt that when the distance between the point of suspension *S* and centre of gravity *G* is *L*, a point *P* corresponding to the centre of oscillation exists on the other side of *G* at the distance $\frac{k_r^2}{L}$, where k_r is the radius of gyration of the body about an axis passing through *G*. Hence the distance between *S* and *P* is

$$L' = L + \frac{k_r^2}{L} = \frac{L^2 + k_r^2}{L} \qquad (7.1)$$

Centre of oscillation is very important for the sports persons using bats (or rackets) for their game. If the ball strikes at the point of centre of oscillation of the bat, it simply rotates by the impact of the ball, but the hand of the player does not feel any impact. This point is also known as Centre of Percussion or Sweet Point.

That is, the centre of oscillation lies at the equivalent length of the simple pendulum (L') from the point of suspension. Since the period of oscillation is same for both these points, we can use them interchangeably. In this experiment you will use Kater's pendulum to obtain the value of gravitational acceleration by taking readings of time periods of oscillations around both these points.

Expected Skills

After performing this experiment, you should be able to:

- understand the construction of Kater's pendulum;
- assemble the apparatus of Kater's pendulum;
- ascertain the centre of gravity of the assembly and balance the pendulum;
- take the readings for time period of oscillations around point of suspension and centre of oscillation; and
- use the observed time periods to calculate the value of g.

The apparatus required for this experiment is given below.

Apparatus required

Kater's pendulum; suspension bracket fixed on the wall, meter scale and stop watch.

Let us now describe the underlying theory of this experiment and construction of the apparatus.

7.2 BASIC THEORY AND CONSTRUCTION OF KATER'S PENDULUM

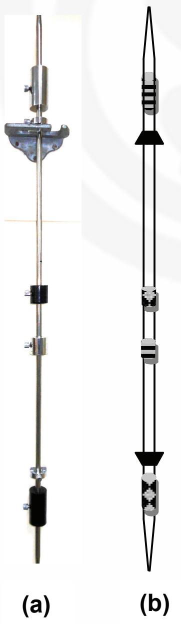


Fig. 7.2: a) Image of Kater's pendulum; b) Schematic diagrams.

Before starting the experiment, you should learn about the construction of the Kater's pendulum. It is a compound pendulum based on the interchangeability of the point of suspension and the centre of oscillation. It consists of about 1 m long metal rod with circular cross-section fitted with two knife edges. Between the ends of the bar and knife edges, two cylindrical weights m_1 and m_2 are fitted. These cylinders are of the same shape but made of different materials viz. wood and metal. This gives rise to asymmetric weight distribution along the rod and hence its centre of gravity G shifts away from its geometrical centre. Two sliding counter weights made up of wood and metal (W_1 and W_2) are fitted between the two knife edges. A photograph of a Kater's pendulum is shown in Fig. 7.2a, while its schematic diagram is given in Fig. 7.2b.

Position of the two knife edges and weights is adjusted in such a way that the period of oscillation of the pendulum about both the knife edges is equal. In this condition, when one knife edge acts as point of suspension, the other one represents the centre of oscillation and the distance between the two knife edges is equal to the equivalent length of the simple pendulum L', whose time period is given by

$$T = 2\pi\sqrt{\frac{L'}{g}} \qquad (7.2)$$

Now, let T_1 and T_2 be the periods of oscillation about the knife edges A_1 and A_2, respectively. If the distance of A_1 from G is L_1 and that between A_2 and G is L_2, then, from Eq. (7.1), we have

$$T_1 = 2\pi\sqrt{\frac{L_1^2 + k_r^2}{L_1 g}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{L_2^2 + k_r^2}{L_2 g}}$$

i.e.,
$$\frac{L_1 g T_1^2}{4\pi^2} = L_1^2 + k_r^2 \qquad (7.3a)$$

and
$$\frac{L_2 g T_2^2}{4\pi^2} = L_2^2 + k_r^2 \qquad (7.3b)$$

With some algebra the acceleration due to gravity is given by (read the margin remark)

$$g = \frac{8\pi^2}{\dfrac{T_1^2 + T_2^2}{L_1 + L_2} + \dfrac{T_1^2 - T_2^2}{L_1 - L_2}} \qquad (7.4)$$

Subtracting Eq. (7.3b) from Eq. (7.3a),

$$\frac{g}{4\pi^2}(L_1T_1^2 - L_2T_2^2) = L_1^2 - L_2^2$$

$$\therefore \quad \frac{4\pi^2}{g} = \frac{L_1T_1^2 - L_2T_2^2}{L_1^2 - L_2^2}$$

Using the method of partial fractions, we get

$$\frac{4\pi^2}{g} = \frac{T_1^2 + T_2^2}{2(L_1 + L_2)} + \frac{T_1^2 - T_2^2}{2(L_1 - L_2)}$$

When the values of T_1 and T_2 are close to each other, the second term in the denominator becomes negligible and we can write

$$g = \frac{8\pi^2(L_1 + L_2)}{T_1^2 + T_2^2} \qquad (7.5)$$

When the Kater's pendulum is properly balanced, we have $T_1 = T_2 \, (= T)$ and get

$$g = \frac{4\pi^2(L_1 + L_2)}{T^2} \qquad (7.6)$$

Now, under the balanced condition, the length between the two knife edges $(L' = L_1 + L_2)$ represents the equivalent length of the simple pendulum.

7.3 EXPERIMENT WITH KATER'S PENDULUM

You have learnt that construction of Kater's pendulum is quite complicated and so it is very important to arrange all its components correctly before starting the measurements using it. You should assemble the apparatus as described below.

7.3.1 Setting up the Apparatus

1. Keep the pendulum in horizontal position on a table and arrange the knife edges (A_1 and A_2) and weights on the bar as shown in Fig. 7.3.

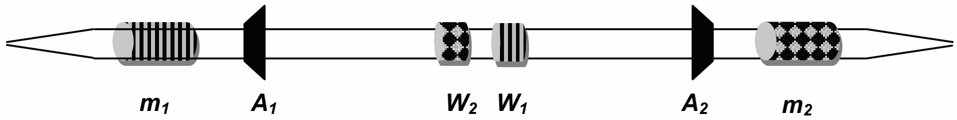


Fig. 7.3: Arranging Kater's pendulum.

Here m_1 and W_1 are wooden weights while m_2 and W_2 are metal weights, which are much heavier than m_1 and W_1.

2. Place the counter weights W_1 and W_2 in the middle of the bar as shown in the Fig. 7.3.

3. Fix the knife edges A_1 and A_2 about 20 cm from both ends of the rod such that their sharp edges face each other.

4. Fix m_1 and m_2 about 15 cm from the two ends of the rod.

5. Now place the meter scale on the table and measure the distance between the knife edges (L') and write it in Observation Table 7.1. This distance will be about 65 to 75 cm depending on the rod length, and L' will be constant during the entire experiment.

7.3.2 Taking Measurement

1. Place this pendulum horizontally on a sharp edge object like metre scale and balance it horizontally to determine the centre of gravity of this assembly. Mark the point (G) where it balances perfectly. This is the **balance** position of the pendulum. Measure the distance between this point and knife edge A_1 as shown in Fig. 7.4 and note it as length L_1 in the Observation Table 7.1 at Sl. No. 1. Also note down the distance between G and A_2 as L_2. Obviously you will have $L_1 + L_2 = L'$.

2. Now place the pendulum assembly vertically on the wall bracket such that it is suspended by knife edge A_1. Make sure that only the knife edge is touching the glass slides of the bracket and no other part of the pendulum is brushing with the bracket on the wall.

3. Now give a gentle oscillation to the pendulum such that it performs simple harmonic motion in the plane parallel to the support wall without touching anywhere.

   Using a stop watch measure the time required for 5 oscillations and note it as T_1 in the Observation Table 7.1 against Sl. No. 1. Calculate time period for single oscillation: $t_1 = \frac{T_1}{5}$.

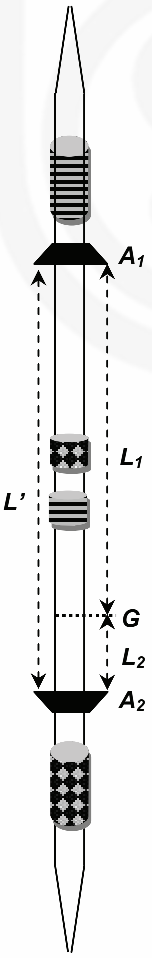


Fig. 7.4: Lengths on Kater's pendulum.

4. Now rotate the pendulum assembly and rest it on knife edge A_2. Again measure the period for 5 oscillations and note it as T_2. Calculate time period of single oscillation: $t_2 = \dfrac{T_2}{5}$.

5. Now remove the pendulum from the bracket and place it on the table horizontally.

6. Move weight W_2 towards A_1 by 1 cm and fix it properly.

7. Obtain the new position of G. Now, measure new L_1 and L_2 as described in step 1 and note these values in Observation Table 7.1 at Sl. No. 2.

8. Repeat steps 2 to 4 and note T_1 and T_2 at Sl. No. 2 and calculate new t_1 and t_2.

9. If the difference between t_1 and t_2 is less than the observations at Sl. No.1, continue to shift W_2 further towards A_1 by 1 cm. If the difference has increased, shift W_2 towards A_2 (in the opposite direction).

10. Repeat steps 2 to 4 to obtain L_1, L_2, T_1 and T_2 and note them at subsequent serial numbers (from Sl.No.3 onwards) in the Observation Table 7.1 and calculate t_1 and t_2.

11. Applying the test given in step 9, decide the direction of movement of W_2 on the bar and repeat the above procedure.

12. You should continue to move W_2 and note t_1 and t_2 until the difference between them is less than 0.01 s ($|t_1 - t_2| < 0.01\,\text{s}$).

13. Now take the readings of T_1 and T_2 for 20 oscillations each and note them in Observation Table 7.2. Calculate t_1 and t_2 by dividing T_1 and T_2 by 20. Check whether the difference between t_1 and t_2 is still less than 0.01s. If not, move W_1 (wooden counter weight) and repeat steps 2 to 10 for 20 oscillations till you obtain a balance position. [Since W_1 is much lighter than W_2 displacing W_1 over larger lengths will affect t_1 and t_2 very slightly and hence it is useful for fine tuning.]

14. When you obtain the balance position for 20 oscillations, repeat the steps 2 to 4 for 50 oscillations and note T_1 and T_2 in Observation Table 7.3. Repeat this measurement 3 times without disturbing the positions of W_1 and W_2 and calculate t_1 and t_2 each time by dividing T_1 and T_2 by 50.

15. Calculate average values of t_1 and t_2 and note them in Observation Table 7.3.

16. Now remove the pendulum from the wall bracket and place it horizontally on a table. Following Step 1 again, obtain the values of L_1 and L_2 and note them below the Observation Table 7.3.

17. Calculate $L_1 + L_2 = L'$. It should be the same value that you had obtained at Point 5 in the Setting up the Apparatus Section (7.3.1) and noted on the top of Observation Table 7.1.

18. Calculate $L_1 - L_2 = L''$.

19. Using the formula given in Eq. (7.4), calculate the value of *g*.

Observations:

Observation Table 7.1: Readings by moving W_2 (metal)

Distance between knife edges = L' = …………….. cm = …………m

Sl. No.	L_1(m)	L_2(m)	T_1 for 5 oscillations suspended from A_1 (s)	T_2 for 5 oscillations suspended from A_2 (s)	$t_1 = \frac{T_1}{5}$ (s)	$t_2 = \frac{T_2}{5}$ (s)
1.						
2.						
3.						
⋮						

Observation Table 7.2: Readings by moving W_1 (wood)

Sl. No.	L_1(m)	L_2(m)	T_1 for 20 oscillations (s)	T_2 for 20 oscillations (s)	$t_1 = T_1/20$ (s)	$t_2 = T_2/20$ (s)
1.						
2.						
3.						
4.						

Observation Table 7.3: Readings for 50 oscillations

Sl. No.	T_1 for 50 oscillations (s)	T_2 for 50 oscillations (s)	$t_1 = T_1/50$ (s)	$t_2 = T_2/50$ (s)	Average t_1 (s)	Average t_2 (s)
1.						
2.						
3.						

Balancing length L_1 (from A_1 to G) = …………….. cm = …………m
Balancing length L_2 (from A_2 to G) = …………….. cm = …………m

Calculations:

$$L_1 - L_2 (= L'') = ... \text{m}$$

$$L_1 + L_2 (= L') = ... \text{m}$$

From the average values in Observation Table 7.3, calculate

$$t_1^2 = \ldots\ldots\ldots\ldots\ldots\ldots \, s^2$$

$$t_2^2 = \ldots\ldots\ldots\ldots\ldots\ldots \, s^2$$

From Eq. (7.4), calculate the acceleration due to gravity,

$$g = \frac{8\pi^2}{\dfrac{t_1^2 + t_2^2}{L_1 + L_2} - \dfrac{t_1^2 - t_2^2}{L_1 - L_2}} \, \text{ms}^{-2}.$$

$$= \frac{8\pi^2}{\dfrac{t_1^2 + t_2^2}{L'} - \dfrac{t_1^2 - t_2^2}{L''}} \, \text{ms}^{-2}.$$

Results:

Acceleration due to gravity,

g = ………… ms^{-2}.

EXPERIMENT 8

STUDY OF THE MOTION OF A SPRING-MASS SYSTEM: DETERMINATION OF SPRING CONSTANT AND ACCELERATION DUE TO GRAVITY

Structure

8.1 Introduction

Expected Skills

8.2 Theory of Spring-Mass System

8.3 Determination of Spring Constant

Static Method of Measuring *k*

Dynamic Method of Measuring *k*

8.4 Determination of the Acceleration due to Gravity

8.1 INTRODUCTION

In the previous experiment, you have determined acceleration due to gravity using Kater's pendulum. We now investigate spring constant and value of acceleration due to gravity, g, using the spring-mass system.

We find many uses of spiral springs in daily life. Springs hold dry cells in proper position in a transistor set and a pocket calculator. Springs are used as shock absorbers in automobiles and railway wagons. You may have also used yourself a bull-worker or seen body-builders using it. Do you know that it essentially consists of springs? In wrist watches, springs control oscillations of the system. In all these cases, the basic difference in the springs being used is in their spring constants. So to decide on the type of a spring for a particular purpose, you must know its spring constant. In a physics laboratory you can determine the value of spring constant, k, by:

i) measuring extension in the spring for a given load (static method), and

ii) determining the period of harmonic oscillations of a spring-mass system (dynamic method).

In this experiment, you will learn the theory of spring-mass system in Sec. 8.2. In Sec. 8.3 you will determine spring constant and obtain the value of g.

Expected Skills

After performing this experiment, you should be able to:

- ❖ measure extension of a spring for a given load and calculate its spring constant (static method);
- ❖ measure the period of oscillation of a spring-mass system for different loads and calculate k (dynamic method);
- ❖ compare the accuracies of static and dynamic methods; and
- ❖ determine the value of g.

The apparatus required for this experiment is listed below.

Apparatus required

A spiral spring, slotted weights in multiples of 100g, stop watch, a laboratory stand and a 50 cm scale

8.2 THEORY OF SPRING-MASS SYSTEM

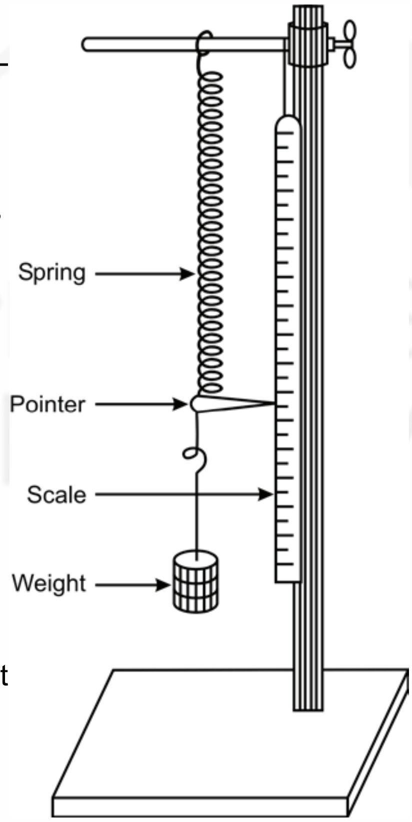


Fig. 8.1: A spring-mass system.

In Experiment 6, you investigated the question: What determines the values of T for a bar pendulum? You may now ask: Can we make similar investigations for a spring-mass system? The answer is in affirmative. In this experiment you will calculate the spring constant of a spring in two different ways: (i) by measuring extension for a given load, and (ii) by measuring the period of harmonic oscillations of a spring-mass system.

Refer to Fig. 8.1, which shows a spring beside a metre scale suspended on the stand. Fix a sharp-tipped pointer (needle) at the lower end of the spring. If you do not get a needle, you can make a pointer of cardboard by cutting it in the shape of a triangle. Then you have to attach its base to the straight end of the spring so that its vertex moves along the scale. This helps in minimising parallax error. Suspend a hanger (which itself is a known weight, equal to any other slotted weight) in the hook of the spring. (Alternatively, you can tie a pan to the lower end of the spring and put weights.) Normally, it is advisable to put an initial load on the hook as it will take care of the kinks in the spring. This implies that the choice of the initial position does not matter.

Stretch the spring by pulling the hanger downwards through a small distance and then let it go. The spring-mass system will execute vertical oscillations. Ensure that the pointer does not stick anywhere and the oscillations are free. Now your apparatus is ready and you can start your experiment. But before you do this, do spend a few minutes in making qualitative observations as to how extension/period changes when the mass is changed within **elastic limits**. This limit will be different for different springs. So you should consult your counsellor before putting a load on the spring.

When we load the spring by putting a weight, a restoring force is set up in the spring due to elasticity. It tends to oppose the applied force and bring the system back to its original state. If extension is small compared to the original length of the spring, the magnitude of restoring force exerted by the stretched spring on the mass is given by Hooke's law:

$$F = -k\,x, \tag{8.1}$$

where x is extension in the spring and k, the spring constant.

When an external force is applied on a body, it tries to retain its shape and size. And as soon as the applied force is removed, the body regains its original state. This property is called **elasticity**. Its maximum limit is called **elastic limit**. If applied force exceeds elastic limit, it produces permanent deformation and the body fails to regain its original shape and size even when the applied force has been removed.

From Eq. (8.1) it is clear that once we know extension as a function of load, k can be calculated easily. It is with this purpose that we attach a pointer to the lower end of the spring. This method of determining k is known as *static method*.

You may now like to know: Is there some other method also for determining k? We can use the so-called *dynamic method*. It is based on observing the period of harmonic oscillations of the spring-mass system.

In Unit 16 of theory course on Mechanics, you have learnt that a spring-mass system executes SHM like a simple pendulum, provided the extension is not large. Another question that comes to our mind immediately is: Does gravity affect the frequency of oscillations? Gravity has no effect on the frequency of oscillations. The period of oscillation is given by

$$T = 2\pi\sqrt{m/k} \tag{8.2}$$

This relation shows that we can determine k by knowing the period of oscillations for a given mass. The value of m will depend on the nature of spring. For a thin spring, m could be a few grams.

8.3 DETERMINATION OF SPRING CONSTANT

As discussed in theory, the spring constant can be determined by two methods viz. static and dynamic. Now we explain the procedure to determine k using these two methods.

8.3.1 Static Method of Measuring *k*

The procedure for measuring k using static method is as follows:

1. Load the spring by putting a weight and record the corresponding equilibrium position of the pointer. Treat this equilibrium position of the pointer on the scale as your initial observation. Record your reading in Observation Table 8.1.
2. Now increase the load in steps by adding equal weights each time.
3. For each load record the position of the pointer.
4. Before taking a reading, you should wait for some time so that the pointer comes to rest.
5. Take at least six observations.

Observation Table 8.1: Extension as a function of load

Sl. No.	Load on the spring (g)	Reading of the pointer on the metre scale (cm)		
		Increasing load	Decreasing load	Mean Reading
1.				
2.				
3.				
4.				
5.				
6.				

To ensure that you are working within the permissible elastic limit, you should record the position of the pointer by unloading the spring in the same steps. Again tabulate your readings in Observation Table 8.1. Do these readings differ from those recorded while loading the spring? If observations for a given weight are nearly the same, both while loading and unloading, you can be sure that you are certainly working within the elastic limit. Note that you have to observe the mean reading of the pointer for a given load.

Now you should plot a graph between the load and the corresponding elongation. Conventionally, we plot the independent variable along the *x*-axis and the dependent variable along the *y*-axis. Which physical quantity will you plot for this experiment along the *x*-axis? Obviously, load should be plotted along *x*-axis. Draw the best fit line through observed points as shown in Fig. 8.2. (For a good steel spring, we expect the graph to be linear.)

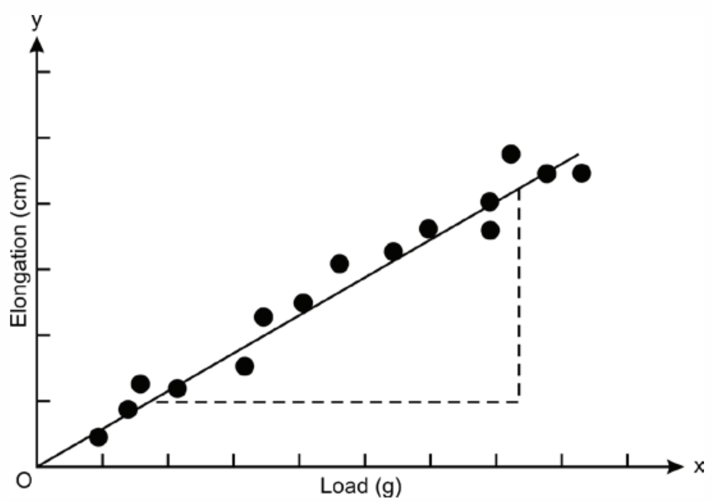


Fig. 8.2: Best fit line through observed points.

If the extension corresponding to mass M is x_0, we can write

$$Mg = kx_0 \quad \text{(i)}$$

Note that we are considering magnitudes only.

Let δ be the elongation corresponding to an additional load m. Then we have

$$(M+m)\,g = k\,(x_0 + \delta) \quad \text{(ii)}$$

From (i) and (ii), we get

$$mg = k\delta$$

$$\text{or} \quad k = \frac{mg}{\delta} = \frac{g}{\text{slope}} \quad \text{(iii)}$$

Suppose that the graph paper is somehow not available in your laboratory. You may then ask: How to calculate k? You will have to use Eq. (iii). Suppose you have taken seven readings. Then calculate extension δ for load difference between readings 4 and 1;5 and 2; 6 and 3; and 7 and 4. Calculate mean value of δ and hence k.

Does your straight line pass through the origin? The inverse of the slope of the straight line is a measure of the spring constant. To calculate the slope, you should use two widely separated points on the straight line. These should be other than observation points. Use g = 9.8 ms^{-2} to calculate k and express your result in SI units.

Error Analysis

Calculate the change in slope of the straight line caused by drawing the lines of maximum and minimum slopes. This gives maximum error in the slope. Using g = 9.8 ms^{-2} calculate the error in the value of k in SI units.

Conclusion: The spring constant of the given spring = ………. ± …..… Nm^{-1}

SAQ 1 – **Static method**

From your graph, calculate the extension for a load of 2 N.

8.3.2 Dynamic Method of Measuring *k*

In this method, you will be required to measure the period of simple harmonic oscillations. You must ensure that oscillations of the system hanging vertically are longitudinal. That is, there should be no lateral oscillations. Otherwise, the motion will not be simple harmonic.

1. Put a load on the hanger and note the position of the pointer on the scale. Take it as the equilibrium position. Now stretch the spring by pulling the hanger slightly downward and then release it. The system will begin to oscillate. In case there is no lateral oscillation, your apparatus is set. Bring it to rest. Also ensure that the spring executes 20-30 oscillations before their amplitude shows visible decrease.

2. Note the least count of the stop watch and record it in Observation Table 8.2.

3. Now set the spring-mass system into oscillations. Allow the first few oscillations to pass so that there is no anharmonic component. Begin your counting through the equilibrium position and simultaneously start the stop watch. Note the time for N, say 30, complete oscillations.

4. To minimise the error in T, it is desirable to take time for 50 or more oscillations. However, you must ensure that the amplitude of oscillations does not decay significantly. Enter your reading in the Observation Table 8.2. Add more weights in the hanger and repeat the procedure at least five times.

5. Tabulate your observations.

6. How does the time period change? As before, the procedure may be repeated by decreasing the load in same steps. Calculate the mean time for each load.

Observation Table 8.2: Measurement of time as a function of load

Least count of stop watch = ……………… s

Number of complete oscillations counted each time (N) =……….

Sl. No.	Load on the spring m (g)	Time for N complete oscillations (s)			Time period $T = \frac{t}{N}$ (s)
		with load increasing	with load decreasing	mean (t)	
1.					
2.					
3.					
4.					
5.					
6.					

Plot T^2 versus m. Draw the best possible straight line as shown in Fig. 8.3. Does it pass through the origin? From the slope of the straight line, you can easily calculate k.

7. Check if this value agrees with that obtained by the static method. The two values should be same or nearly equal. (In case you get to know the standard value of k for the material of spring from your counsellor or a book, you can judge whether the dynamic method is more accurate than the static method or not.)

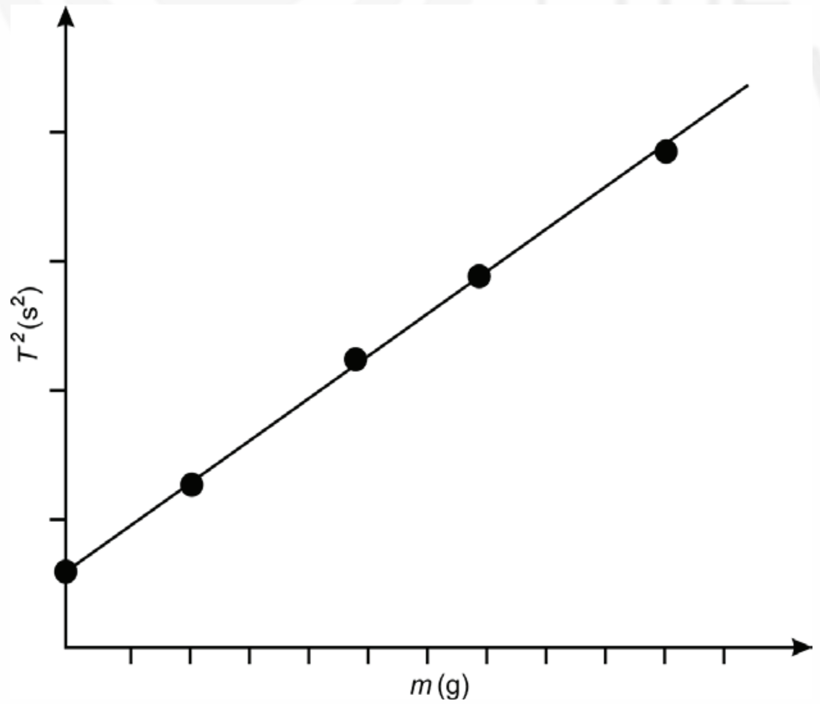


Fig. 8.3: Expected plot of T^2 versus m.

As before, you can compute error in k by drawing lines of maximum and minimum slopes. What is the relative change in the value of k?

Result: Spring constant of the given spring = ………… ±……………Nm^{-1}

SAQ – 2 **Dynamic method**

i) Extrapolate the graph between T^2 and m backward and interpret the intercept.

ii) Use your graph to determine T for a load of 3N.

8.4 DETERMINATION OF THE ACCELERATION DUE TO GRAVITY

We can use these set up to determine the value of acceleration due to gravity, g.

Firstly, we can take Eq. (8.2) and rewrite this equation by taking square on both sides as

$$T^2 = 4\pi^2 \frac{m}{k} \tag{8.4}$$

From Fig. 8.3, which shows the graph between T^2 (on y-axis) and m (on x-axis), the value of slope obtained is $4\pi^2 / k$ or $k = \frac{4\pi^2}{(\text{slope})}$. After obtaining the value of k using dynamical method, the value of g can be calculated using Eq. (iii) given in the margin remark of Sec. 8.3.1 $k = \frac{mg}{\delta} = \frac{g}{\text{slope}}$ or $g = k \times \text{slope}$

Value of g = …………………………

EXPERIMENT 9

DETERMINATION OF FREQUENCY OF TUNING FORK USING SONOMETER

Structure

9.1 Introduction
Expected Skills
9.2 Stationary Waves in a Stretched Wire
9.3 Variation of Wavelength with Tension
9.4 Variation of Wavelength with Mass per Unit Length
9.5 Relation between Wavelength and Frequency

9.1 INTRODUCTION

In the previous experiment, you have learnt to determine the spring constant and value of acceleration due to gravity using a spring-mass system. We now determine the frequency of a tuning fork using sonometer.

You all must have enjoyed the pleasing music produced by string instruments like sitar, violin, guitar, *ektara*, etc. Do you know how stringed instruments produce music? When the string of such an instrument is plucked, bowed or struck, it begins to vibrate and produces sound. The quality of sound depends on the frequency of vibration of the stretched string. Now the question arises: What factors determine the frequency of vibration of a string? How are these factors related to frequency? In this experiment, you would discover answers to such questions.

Tuning a given musical instrument with another means adjusting the frequency of the given instrument so that it is the same as that of the other one.

You may have observed that in an orchestra, a violinist tightens or loosens the pegs of the instrument while tuning with other musicians. (As the peg is tightened or loosened, a portion of the string is either wound or unwound around the peg). As a result, tension in the string changes. This suggests that the frequency produced by the string of the violin depends on the tension in it. Can you think of other parameters that may influence the frequency of vibration of a string? What happens if you take strings of same material having different thicknesses or strings of different materials but same thickness? Well, we expect that the frequency of vibration of the string in each case should differ. This means that the mass per unit length of the string also influences its frequency of vibration.

You may have seen a *veena*. In this musical instrument, strings of unequal lengths are tied between two fixed ends. You may have also seen that once a musician has tuned the instrument, she moves her fingers along its string to produce music. In this way, she varies the vibrating length in order to produce different notes. This suggests that the frequency of vibration of the string depends on its vibrating length as well. We know that the length of the vibrating segment of the string is related to the wavelength of the stationary waves set up in it. Hence, we expect that there exists a definite relationship between the wavelength and frequency.

The aim of this experiment is to know how frequency of vibrations of a stretched string depends on tension, mass per unit length and its vibrating length. In this case, any change in the frequency can be attributed to the change in that particular parameter.

It is possible to set up waves of known wavelength in a wire. But it is more convenient to make a wire vibrate with a known frequency. So we would discover the effect of tension and mass per unit length of the wire on the wavelength, keeping the frequency constant. Therefore, we would like you to do this experiment in three parts. In the first part, you should investigate how the wavelength changes with tension in the wire while the frequency of vibration of the wire and its mass per unit length are kept fixed. In the second part, you will investigate how the wavelength varies when wires of different thicknesses (but same material) or different materials (but same thickness) are used. That is, you will learn how wavelength varies with mass per unit length of the wire when tension in the wire and frequency are not changed. In the third part, you will establish the relation between frequency and wavelength, keeping the tension and mass per unit length of the wire fixed.

Expected Skills

After performing this experiment, you should be able to:

- ❖ set up stationary waves in a stretched string;
- ❖ investigate the dependence of wavelength of stationary waves on tension in a string and its mass per unit length;
- ❖ establish the relation between wavelength and frequency; and
- ❖ obtain the expression for velocity of transverse waves on a string.

The apparatus required for the experiment is listed below.

Apparatus required

Four iron wires of different thicknesses (Alternatively 4 wires of different magnetic materials), sonometer, hanger, slotted weights, an electromagnet with a 6 volt a.c. transformer, six tuning forks of known frequencies, rubber pad, metre scale, screw gauge, a chemical balance and a weight box.

9.2 STATIONARY WAVES IN A STRETCHED WIRE

Setting up of Stationary Waves

The measurement of tension (T) and mass per unit length (μ) of a stretched wire are rather routine exercises. But to make a precise determination of wavelength, we set up *stationary waves*. Stationary waves are formed when two identical progressive waves moving in opposite directions are made to superpose. The stationary waves do not move with time in either direction. (For this reason, they are also sometimes referred to as *standing waves.*) From your school physics, you will recall that stationary waves can be produced in air columns as well as stretched strings. Here we intend to set up stationary waves in a sonometer wire.

A wave which transports energy as it propagates in space is said to be progressive. In a stationary wave, no energy is transported.

Refer to Fig. 9.1. It shows a sonometer, which consists of a hollow wooden box with circular holes, a peg at one end and a pulley on the other. One end of a wire is fixed to the peg and the other end, passing over a smooth pulley, carries a hanger. (In place of hanger, you can also use a pan.) By placing weights on the hanger, the string can be stretched. The wire passes over two bridges B_1 and B_2. While performing experiments with a sonometer, the string is made to vibrate in **unison** with the source of sound, which may be a tuning fork or an electromagnet. To achieve this, the vibrating length of the wire between the bridge is adjusted by sliding the bridges between the peg and the pulley. This condition (of unison) is said to be ensured when a V-shaped paper rider placed in the middle of the wire between the bridges falls down.

The sonometer wire is said to vibrate in *unison* with the source of sound when the natural frequency of the wire equals the frequency of the source.

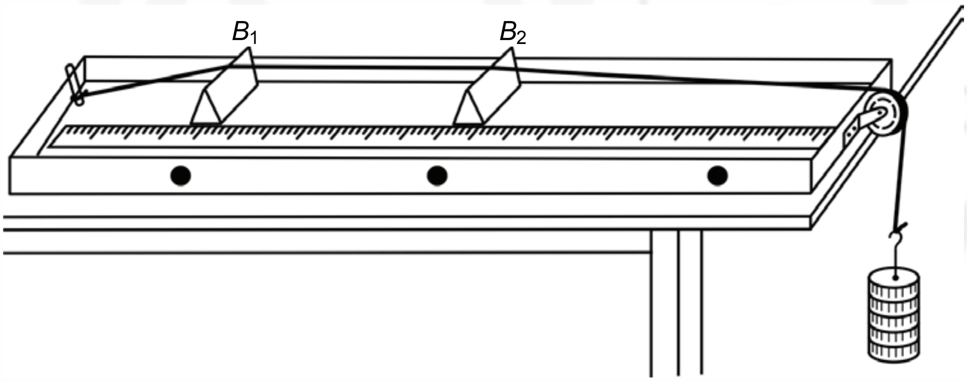


Fig. 9.1: Stationary waves in a stretched string of a sonometer.

In your school physics, you have learnt that when a vibrating tuning fork is placed on the sounding board of the sonometer, the air inside the sonometer begins to vibrate. It makes the wire to execute forced vibrations leading to formation of transverse waves. In the region B_1 B_2, these transverse waves are reflected at the fixed points B_1 and B_2. As a result, we obtain a set of incident and reflected waves travelling in opposite directions. Their superposition gives rise to stationary waves. The wire between the bridges then vibrates in one or more well-defined segments, as shown in Fig. 9.2. Note that there are some points at which the wire remains motionless at all times. On the other hand, at some other points, the waves reinforce strongly and the wire vibrates vigorously. The points corresponding to zero amplitude of vibration are called nodes (N), whereas points with maximum amplitude are called antinodes (A). The simplest mode of vibration occurs when the string vibrates in a single loop

The vibrations are said to be forced vibrations when a body vibrates with the frequency of the applied periodic force. In this condition, the energy fed from outside equals the energy lost by the body.

(Fig. 9.2a). The frequency of vibration corresponding to this mode is known as the **fundamental frequency** of vibration.

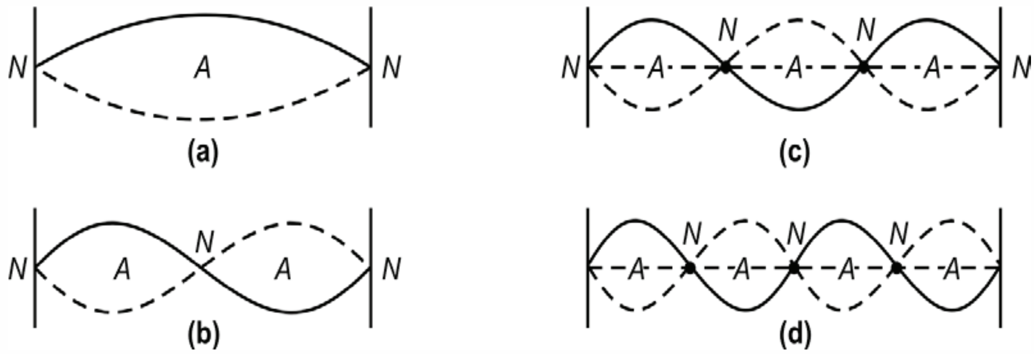


Fig. 9.2: Stationary waves set up in a wire fixed at both ends.

9.3 VARIATION OF WAVELENGTH WITH TENSION

In this part of the experiment, you have to keep mass per unit length (μ) of the wire and its fundamental frequency of vibration constant. Working with a wire of uniform cross section ensures constancy of μ. To achieve the latter, you can use either a tuning fork or an electromagnet. We advise you to use an electromagnet, if available, because it can make the wire execute sustained vibrations.

In case you are not provided an electromagnet, choose a tuning fork of known frequency. (You may also discuss with your counsellor.) As you know, we have “musical ears”. You can get close to the condition of unison using your ears. To do this, strike the tuning fork on the rubber pad and hold it near your left ear. Strike the sonometer wire between the bridges with your finger and hold your right ear near the sonometer wire. As long as the frequencies produced by the tuning fork and sonometer wire are not in unison, you will hear two distinct sounds with different frequencies. But by adjusting the position of bridges, gradually you can attain near unison condition. Next strike one of the prongs of the tuning fork with a rubber pad and press the stem of the fork on the sounding board of the sonometer. Do not touch its U-part. (If you do so, the vibrations of tuning fork will die rapidly.) You will observe that the wire begins to vibrate resulting in stationary waves. The paper rider placed in the middle of the wire will fall when it resonates with the turning fork.

While working with a tuning fork, you may observe that vibrations may not be sustained for long. Then you should strike the prong of tuning fork again with the rubber pad and place it on the sounding board to determine the resonating length for each load. Moreover, since the energy supplied by the tuning fork to the vibrating wire is many-times less than that given by the electromagnet, the wire will not vibrate vigorously. Therefore, in this case, you have to rely more on the paper rider, which falls off or vibrates vigorously when unison occurs.

The experimental arrangement for generating stationary waves in a sonometer wire using an electromagnet is shown in Fig. 9.3. Connect the electromagnet to a 6 V transformer and place it near the middle of the wire. When the electromagnet is connected to a source of AC power supply, the core of the electromagnet will be magnetized twice with opposite polarities in each cycle. As a result, the sonometer wire gets attracted towards the electromagnet twice in each cycle and begins to vibrate. Since the frequency of AC is 50 Hz, the wire will vibrate with a fixed frequency of 100 Hz.

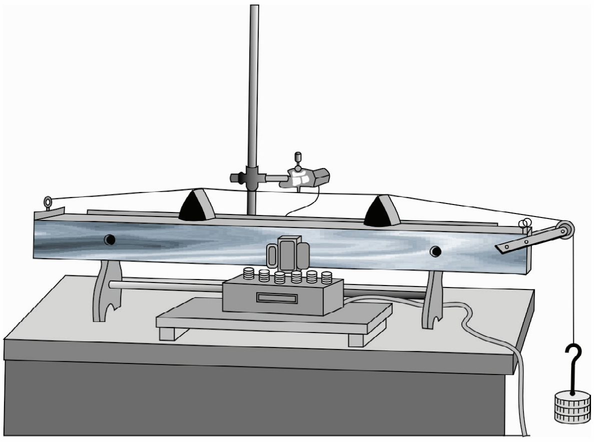


Fig. 9.3: Experimental arrangement for generating transverse stationary waves in a sonometer wire using an electromagnet.

SAQ – 1 Inducing forced vibrations

Suppose that the electromagnet is connected to a source of direct current. Will the wire vibrate? If so, what will be its frequency of vibrations?

Stretch the wire by putting an appropriate weight on the hanger. (You should consult your counsellor in determining this.) If the mass of the hanger is m kg and a weight of M kg is used in stretching the wire, the tension in the wire will be $T = (M + m)g$ N where g is acceleration due to gravity.

Keep the bridges B_1 and B_2 on the sonometer at the largest possible separation and switch on the current through the electromagnet. The wire will begin to vibrate. This means that the apparatus is now in working condition and you can begin your investigations.

Your objective is to determine the length of the wire for which the sonometer vibrates in the fundamental mode. This happens when the wire vibrates in a single loop with maximum amplitude. This length corresponds to the separation between two consecutive nodes and is equal to half the wavelength of the stationary wave in the wire.

When you switch the current on, the wire is supposed to vibrate with a frequency of 100 Hz. But you may not see the wire vibrate at all. Do you know why? This is likely to happen if the length of the wire between B_1 and B_2 is much different from that corresponding to the fundamental frequency and the amplitude of forced vibration set up in the wire is extremely small. So you have to adjust the length of the vibrating wire. To do this, keep one of the bridges (say, B_1) fixed and move the other bridge (B_2) towards it slowly. What do you observe? Does the amplitude of vibration increase? If so, continue to decrease the vibrating length of the wire by moving the bridge B_2 closer to B_1 until the amplitude of vibration becomes maximum. You will then clearly see that the wire is vibrating in a single loop of significant amplitude. If you place a paper rider gently in the middle of the wire now, it will be thrown off. Note the weight and the corresponding length between B_1 and B_2 by noting their positions on the metre scale attached to sonometer board. Record the readings in Observation Table 9.1. Next, move the bridge B_2 closer to B_1 by a small distance (2–3 cm). What do you observe? Does the amplitude of vibration change? If so, the frequency of the vibrating wire is not 100 Hz. Then, slowly move the bridge B_2 away from B_1 and locate the position where wire vibrates in unison again. You should repeat this act 3-4 times for a given tension to minimize the error in your observation. You will also hear maximum sound when the vibrating wire is in unison with the forced frequency.

Now, you change the tension in the wire by adding weights of 0.2 kg or 0.5 kg in equal steps and measure the resonating length of the wire in each case following the procedure outlined above. You will observe that the resonating length increases with increasing load. Enter your data for each step in Observation Table 9.1. You should not load the wire beyond its elastic limit. (Consult your counsellor to know this value).

To check that you are working within the permissible range of tension, you should repeat the above procedure by unloading the wire in the same equal steps and measure the resonating length of the wire. Tabulate each reading. Do these lengths differ from those obtained for corresponding tension while loading the wire? We expect these to be almost the same.

Observation Table 9.1: Dependence of wavelength on tension

Frequency of vibration of the wire =…………………………Hz

Least count of metre scale =…………………………..….cm

Mass of the hanger (m) =…………………………kg

Sl. No.	Weight placed on hanger, M (kg)	Tension $T = (M+m)\,g$ (N)	Resonating length of the wire between the bridges B_1 and B_2 (m)				Mean resonating length for a given load, $\ell = \frac{\overline{L}_1 + \overline{L}_2}{2}$ (m)	Wave-length $\lambda = 2\ell$ (m)
			Load increasing		Load decreasing			
			L_1(m)	Mean Value $\overline{L}_1$	L_2(m)	Mean Value $\overline{L}_2$		
1.								
2.								
3.								
4.								

From the Observation Table 9.1 you will observe that λ changes with T. Mathematically, we can write

$$\lambda = f(T)$$

Can you quantify this relation exactly by looking at your observations? Probably you cannot. To discover the exact relationship between λ and T, we write

$$\lambda \propto T^{a}$$

or

$$\lambda = k_1 T^{a} \qquad (9.1)$$

where k_1 is constant of proportionality and a is another constant.

Taking logarithms on both the sides, we get

$$\log \lambda = \log k_1 + a \log T. \qquad (9.2)$$

Now, take a log-log graph and plot λ along y-axis and T along x-axis. You will obtain a straight line. Its intercept on the y-axis is a measure of constant of proportionality and the slope of the straight line gives the value of a. Calculate the slope by using two well separated points. We expect the value of a to be one-half.

If log-log graph papers are not available in your laboratory, you should calculate and plot λ versus $T^{½}$, T, T^{2}, etc. The graph which gives a straight line will correspond to Eq. (9.1).

So we can write Eq. (9.1) as

$$\lambda = k_1 \sqrt{T} \qquad (9.3)$$

SAQ 2 – Variation in wavelength with tension

Plot a graph between λ and $T^{1/2}$. Choose the points corresponding to T_1 = 64 N and T_2 = 324 N to calculate the value of wavelength from your graph.

9.4 VARIATION OF WAVELENGTH WITH MASS PER UNIT LENGTH

To investigate the dependence of wavelength on mass per unit length of the wire, take four wires of different thicknesses but of the same material. For each wire, you first determine the mass per unit length (μ). To do so you have to weigh each wire in a physical balance and measure the corresponding lengths. The ratio (m/ℓ) will give you μ. For more precise work, you should measure their diameters (d) using a micrometer screw gauge. Note its least count and observe whether or not there is any zero error. Measure the diameter at several places. In this way you can account for the inhomogeneities, if any, in the wire. Record your readings in Observation Table 9.2(a). Calculate the mass per unit length by the relation $\mu = \dfrac{\pi d^2}{4}\rho$, where d is the mean diameter of the wire and ρ is the density of the material.

Densities of some typical metals

Material	Density ($\times 10^3$ kg m^{-3})
Iron	7.86
Steel	8.03
Nickel	8.912
Copper	8.96
Aluminium	2.698

Table 9.2 (a): Determination of mass per unit length of a wire.

Least count of the micrometre screw gauge = ……cm

Sample wire	Diameter (cm)				Mean diameter d (m)	Density ρ (kg m^{-3})	Mass per unit length of the wire μ(kg m^{-1})
	Obs. No.	Main scale reading	Circular scale reading	Total reading			
A	(i)						
	(ii)						
	(iii)						
B	(i)						
	(ii)						
	(iii)						
C	(i)						
	(ii)						
	(iii)						
D	(i)						
	(ii)						
	(iii)						

In this part of the experiment, you have to keep the tension in the wire constant. To do this, place a weight 2 kg, say, on the hanger. Do not change this weight during this part of the experiment. Now, following the procedure given in Sec. 9.3, determine the distance between the bridges B_1 and B_2 so that the wire vibrates in one loop with the maximum amplitude. Measure the distance and record it in Observation Table 9.2(b).

Repeat this procedure for other wires, keeping the tension in the wire constant. Record your readings in Observation Table 9.2(b).

Observation Table 9.2(b): Dependence of wavelength on mass per unit length

Frequency of tuning fork/electromagnet = …………Hz

Tension in the wire = …………N

Sl. No.	Mass per unit length μ (kg m^{-1})	Length corresponding to unison ℓ (cm)		Mean value of ℓ (cm)	Wavelength $\lambda = 2\ell$ (m)
1.		(i)			
		(ii)			
		(iii)			
2.		(i)			
		(ii)			
		(iii)			
3.		(i)			
		(ii)			
		(iii)			

Does λ change with μ? To quantify this dependence, we write

$$\lambda = k_2 \mu^b, \tag{9.4}$$

where k_2 is constant of proportionality and b is another constant.

Taking logarithms on both the sides, we get

$$\log \lambda = \log k_2 + b \log \mu$$

If you plot λ versus μ on a log-log graph, you will obtain a straight line. Is the slope of the straight line positive or negative? A negative value signifies that as μ increases, λ decreases. The slope of the straight line gives us the value of the exponent b. We expect b = – 0.5. (Discuss your result, if there is significant deviation from the quoted value, with your counsellor.) Thus we can write

$$\lambda = k_2 \mu^{-1/2} \tag{9.5}$$

On combining the results contained in Eqs. (9.3) and (9.5), we obtain

$$\lambda = k\left(\frac{T}{\mu}\right)^{1/2} \tag{9.6}$$

where k is a constant of proportionality.

SAQ 3 – Dependence of wavelength on mass per unit length

i) How would the result of Eq. (9.6) be influenced if the wire stretched on the sonometer were hollow?

ii) Suppose you have adjusted the length of the string (of steel) in unison with a tuning fork. Now you replace the string with a similar one of nickel. Will the same length of the string be in unison with the fork? Why?

iii) From the graph obtained by plotting log λ versus log T from the data recorded in Observation Table 9.1, calculate the intercept on y-axis. How is it related to mass per unit length of the wire? Compare this value with the value estimated from its radius and density.

9.5 RELATION BETWEEN WAVELENGTH AND FREQUENCY

To establish the relation between wavelength and frequency for a given wire, the tension in the wire is kept fixed. To vary the frequency, you would require a set of tuning forks of different frequencies. Obviously, an electromagnet will not be appropriate for this part of your investigations because it makes the wire to vibrate with only one frequency.

To begin with, stretch the wire with an appropriate load, 2 kg weight, say. Now, out of the set of tuning forks, select the tuning fork with the lowest frequency. Keep the bridges B_1 and B_2 maximum distance apart on the sonometer. Now, as discussed in Sec. 9.3, keep B_1 fixed and shift B_2 to adjust the distance between the bridges so that the wire vibrates in one single loop of maximum amplitude. This means that the wire and the tuning fork are in unison. Measure the length and record it in the Observation Table 9.3.

Keeping the tension fixed, repeat the procedure for other tuning forks. Measure the length each time and record it in Observation Table 9.3.

Observation Table 9.3: Dependence of wavelength on frequency

Tension in the string =…………N

Sl. No.	Frequency of the tuning fork f (Hz)	Length corresponding to unison (m)			Mean length ℓ (m)	Wavelength $\lambda = 2\ell$ (m)
		(i)	(ii)	(iii)		
1.						
2.						
3.						
4.						
5.						

How does wavelength of stationary waves depend on the frequency of tuning fork (and hence fundamental frequency of the string for a fixed tension)? We expect the wavelength to decrease as frequency increases. To quantify this dependence, we express it mathematically as

$$f = k_3\, \lambda^c, \tag{9.7}$$

where k_3 is a constant of proportionality and c is some other constant.

Now, if you plot f versus λ on a log-log graph paper you should obtain a straight line. From the slope, you can calculate the value of c. We expect the value of c to be -1. What is your result?

Also from the intercept on the y-axis, you can calculate ln k_3 and hence k_3. Compare this value of k_3 with the ratio $\sqrt{T/\mu}$ for this wire. Are the two values same? Theoretically, they should be. What does it suggest? It implies that frequency and wavelength of stationary waves on a string are connected by the relation

$$f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}\,. \tag{9.8}$$

The dimensions of the product $f\lambda$ are those of velocity (ms^{-1}). From this you can conclude that the velocity of stationary waves in the stretched string is given by $v = \sqrt{\frac{T}{\mu}}$.

Now you may like to attempt on SAQ.

SAQ 4 – Relation between wavelength and frequency

What will be the change in frequency if the unison length of the string between the bridges is doubled?

EXPERIMENT 10

STUDY OF LISSAJOUS FIGURES USING A CATHODE RAY OSCILLOSCOPE

Structure

10.1 Introduction
Expected Skills
10.2 Familiarisation with a Cathode Ray Oscilloscope
10.3 A Function Generator
10.4 Determination of Phase Difference using Lissajous Figures Method
Generation of Phase Difference Signals
Phase Difference Calculation using Lissajous Figures
10.5 Lissajous Figures of Unequal Frequency Sinusoidal Waves

10.1 INTRODUCTION

While studying oscillations you have learnt that the motion of a body subjected to number of simultaneous oscillations can be explained on the basis of superposition principle. If the body is subjected to harmonic oscillations in two mutually perpendicular directions, the path traced by the resultant motion of the body gives rise to **Lissajous figures**. The shapes of these Lissajous figures are determined by the amplitude, frequency and phase relationships between the two mutually perpendicular oscillations applied simultaneously on the body.

In the Physics laboratory you can observe these Lissajous figures using a Cathode Ray Oscilloscope (CRO in short). It is a basic but an important and versatile instrument used in all physics, electronics and electrical engineering laboratory.

Using a CRO, you can measure important characteristic parameters of a signal like voltage amplitude, frequency, period and shape of the waveform. On a CRO screen, a luminous spot enables us to study the instantaneous value of input voltage. For this reason, an oscilloscope can also be viewed as a plotter or a recorder.

In this experiment, you will learn the basic functions of an oscilloscope and use it to study the Lissajous figures by applying two sinusoidal signals to its *x* and *y* inputs and obtain their phase relationship. You will also learn about the function generator required to generate the sinusoidal waves.

Expected Skills

After performing this experiment, you should be able to:

- ❖ understand the functions of various controls on the front panel of the CRO;
- ❖ display a waveform/signal on the screen of the oscilloscope;
- ❖ observe Lissajous figures by applying two sinusoidal waveforms; and
- ❖ calculate the phase difference between two sinusoidal waveforms.

The apparatus required to perform this experiment are given below.

Apparatus required

Dual beam/trace CRO, 2 function generators, resistors (1 k Ω), capacitor (0.5 μF), resistance box (100 Ω – 10 k Ω), bread board, connecting wires, tracing paper.

10.2 FAMILIARISATION WITH A CATHODE RAY OSCILLOSCOPE

Before using a CRO, you must get familiar with its working and the functions of various control knobs on its front panel. CRO is essentially an assembly of a cathode ray tube (CRT) and some specific electronic circuits. CRT is the major component of a CRO. It produces a sharply focussed high speed electron beam, which can be moved on the screen using appropriate voltages for deflection. CRO front panel consists of CRT screen and some knobs to control its function. We have listed the functions of these knobs at the end of this section.

Fig.10.1 shows a schematic diagram of a CRT. It is an evacuated glass envelop with the following essential components:

- an electron gun;
- deflection plates; and
- a fluorescent display screen.

The electron gun has following parts:

- a heater or a cathode that emits electrons;
- a control grid to regulate the amount of current;
- a focusing electrode to produce pencil-like electron beam; and
- accelerating and pre-accelerating electrodes to provide high velocity to electrons, which, on striking the screen, may cause secondary emission.

The deflection assembly comprises of a set of vertical and horizontal plates separated at a distance. The CRT screen has a fluorescent material such as ZnS which emits light when electron strikes on it. In a CRO, the electron beam emitted by the electron gun undergoes deflection before striking the screen.

The deflection assembly comprises of a set of vertical and horizontal plates separated at a distance. The CRT screen has a fluorescent material such as ZnS which emits light when electron strikes on it. In a CRO, the electron beam emitted by the electron gun undergoes deflection before striking the screen.

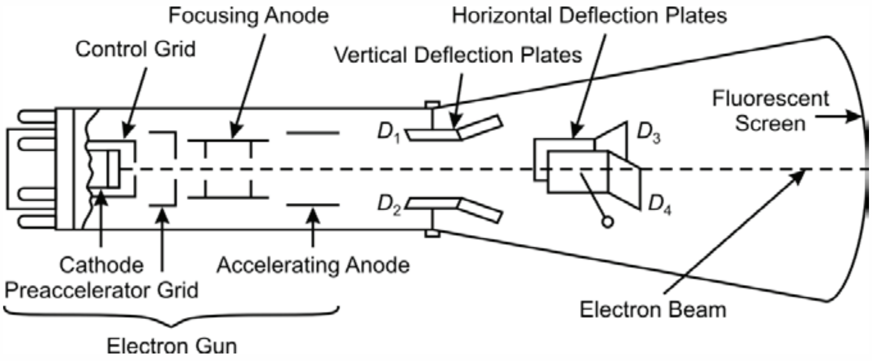


Fig. 10.1: Schematic of a CRT

Since electrons are charged particles, deflection of electron beam can be effected either electrostatically or magnetically. In most of the oscilloscopes, the deflection of the beam is generally caused electrostatically. You may note that the potential applied across the horizontally placed plates D_1 and D_2 would deflect the beam vertically, whereas a potential applied to vertically arranged plates D_3 and D_4 would deflect the beam horizontally. Further, the magnitude of the deflection is proportional to the voltage applied across the deflection plates. In a typical CRT with display screen of about 10 cm, under ordinary conditions, a deflection of about 2.5 cm could be obtained for a potential of about 100V. In the real situation, since the amplitude of the signal we measure with a CRO is well below 100V, we need to amplify the signal in order to cause appropriate deflection of the beam on the CRT screen. Therefore, deflection amplifiers are provided for each pair of deflection plates.

Without going into the details of internal circuits in the CRO, it is sufficient here for you to remember that **for an oscilloscope to display the variation of an electrical signal in the vertical direction as a function of time, a voltage varying linearly with time such as a saw-tooth wave called sweep will have to be applied on the horizontal deflection plates**.

To provide a more stable trace on the oscilloscope, an additional feature in the form of a *trigger* is provided. While using a trigger, the CRO pauses in each cycle when the sweep reaches extreme right side of the screen and retraces back to the left hand side of the screen. Then it waits for a specified event before starting the next trace. The trigger event is usually the input waveform reaching some user-specified threshold voltage in a specified direction (going positive or negative).

For proper operation of an oscilloscope, all the controls are mounted on the front panel. Fig.10.2 depicts the location of various controls on the front panel of a typical general purpose dual beam oscilloscope. In such a CRO, two signals can be viewed simultaneously on two separate channels. We may add here that the location of different controls can vary from one manufacturer to another.

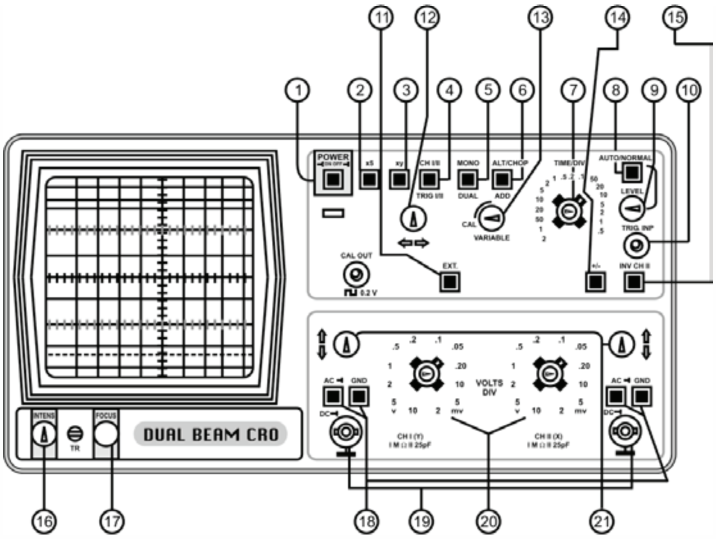


Fig. 10.2: Schematic of front panel of a dual beam general purpose CRO.

Table 10.1 describes the function of each control shown in Fig. 10.2.

Table 10.1: Controls on CRO front panel

No.	Control	Function
1.	Power	Turns mains power on/off.
2.	$\times$ 5	When pressed gives five times magnification of the signal amplitude.
3.	*X-Y*	It cuts off the time base fed to the horizontal plates when pressed in and allows access to the horizontal signal fed through CH-II. It is used for *X-Y* display.
4.	CH-I/CH-II/ Trig I/Trig II	It selects and triggers CH-I when it is out. On pressing it in, it selects and triggers CH-II.
5.	Mono/Dual	A switch to select the single/dual beam operation.
6.	Alt/Chop/Add	It selects alternate or chopped in DUAL mode. If mono is selected, it enables addition or subtraction of signals on two channels.
7.	Time/Div.	It selects time base speeds.
8.	AUTO/NORM	AUTO mode enables trace when no signal is fed at the trigger input. In NORM position, the trigger level can be varied using LEVEL control.
9.	LEVEL	It allows setting of the trigger level between peak-to-peak amplitude of the input signal.
10.	TRIG IN	A socket that is used to feed external trigger signal in EXT mode.
11.	EXT	Switch that allows External triggering signal to be fed from the socket marked TRIG IN.
12.	*X*-POS	This knob controls the horizontal position of the beam trace.

No.	Control	Function
13.	VAR	Controls the time base speed in between two steps of TIME/DIV switch.
14.	+/-	This switch selects the slope of triggering.
15.	INV CH.II	This switch when pressed inverts the signal at CH.II.
16.	INTENS	It controls the trace brightness.
17.	FOCUS	It controls the sharpness of the trace.
18.	DC/AC/GND	Coupling switch for each channel. In AC mode, the signal in coupled through 0.1μF capacitor.
19.	CH-I (*Y*) and CH-II (*X*)	BNC connectors serve as *Y*-input connections for CH-I and CH-II. CH-II input connector also serves as Horizontal external signal on using *X*-*Y* control.
20.	Volts/Div	A switch to select the sensitivity of each channel.
21.	*Y*-Pos I and II	Controls for vertical deflection of trace for each channel.

You must carefully read and understand the function of each control. Then, you should see for yourself how some of the basic controls on the front panel affect a given trace. For this, first switch *on* the oscilloscope by power switch and obtain a horizontal line on the CRO screen (In case of a dual beam / trace oscilloscope, you should obtain two straight lines.) You need not make any connections to the vertical input sections at this stage. Now adjust the controls listed in Table 10.2 and record your observations.

Table 10.2: Functions of some basic controls on CRO front panel

Control	Observed Effect
Intensity	--
Focus	--
Y-position	--
X-position	--

The display area (front panel) of the CRT is marked with a centimeter-scale grid and each centimeter is called **division (Div)**. Each division is further divided into 5 parts; hence the smallest length that can be measured on the screen is 2 mm. For the controls of time base (Time/Div) and voltage sensitivity (Volts/Div), the selected range value corresponds to 1 cm on the display area. [For example, 0.5 ms/div means on the time (horizontal) axis, 0.5 milliseconds are mapped over 1 cm length and 1 V/div means on vertical axis 1 volt amplitude corresponds to 1 cm height on the y-scale].

After getting familiarised with the CRO, now we will discuss in brief about another apparatus called a function generator, which you will be using in this experiment.

10.3 A FUNCTION GENERATOR

You can use an oscilloscope to measure both dc-voltage and time varying voltage. To generate time varying voltage, you need a general purpose function generator, which can generate sinusoidal, triangular and square waveforms with adjustable frequency and amplitude. The function generators are available in either analogue or digital versions. Fig. 10.3 shows the front panel of a typical analogue function generator.

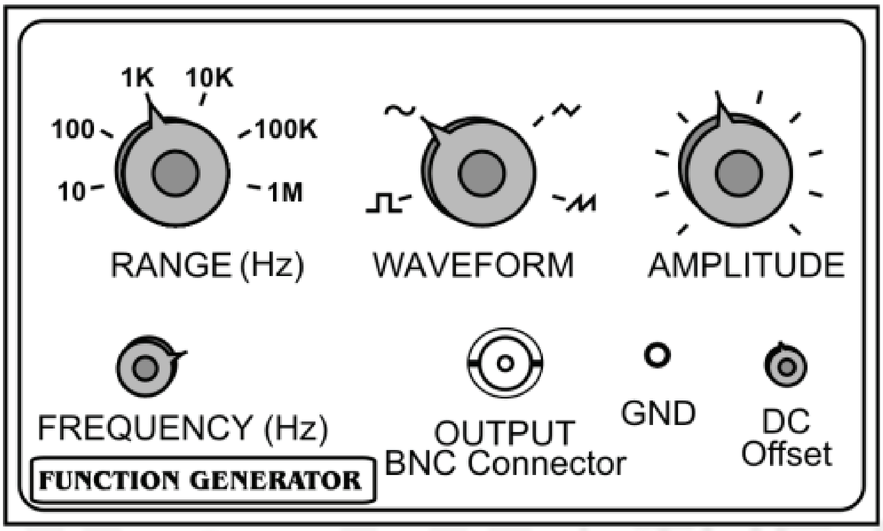


Fig. 10.3: Front panel of a typical analogue function generator.

A function generator usually has control knobs listed in Table 10.3.

Table 10.3: Controls of a typical function generator

No.	Control	Function
1.	WAVEFORM (OR FUNCTION) SELECTOR	Type of waveform/signal: a square wave, sinusoidal, triangular or saw-tooth waveform selection switch
2.	RANGE (Hz)	Frequency range selection switch (10-100-1k-10k-100k-1M) Hz
3.	FREQUENCY	Frequency adjustment knob
4.	AMPLITUDE	Amplitude adjustment knob
5.	OFF SET	DC voltage can be added to the ac signal
6.	OUTPUT	BNC terminal giving out generator signal

10.4 DETERMINATION OF PHASE DIFFERENCE USING LISSAJOUS FIGURES METHOD

You can use an oscilloscope to determine the phase difference between two signals of same frequency by Lissajous pattern method. Let us now learn to obtain signals with **phase difference** using simple electronic circuits.

10.4.1 Generation of Phase Difference Signals

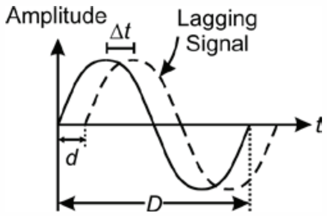


Fig. 10.4: Phase difference between two sinusoidal signals.

Fig. 10.4 shows two sinusoidal waveforms, which have identical time period, that is, they are of equal frequency. However, you must have noticed that they cross the mean position at different times. This time difference multiplied by angular frequency is called the phase difference between the two waves.

Here we choose one signal as a reference, that is, with zero-phase angle. Therefore, the signal being compared is said to be leading by an angle θ if it is to the left of the reference signal and lagging if it is to the right of the reference signal. The lead indicates positive value of phase while lag indicates negative value of phase.

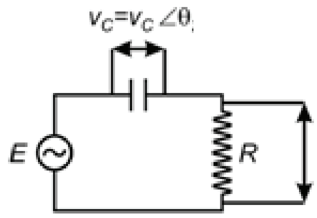


Fig. 10.5: Phase shifting circuit.

In order to obtain two sinusoidal waves of equal frequency but differing in phase, you should use the circuit shown in Fig.10.5. It is an *R*-*C* circuit, and you may recall from your +2 physics course that the current *i* at any instant would lead the applied voltage *E*. Here the voltage v_R is in phase with *i* and the voltage across the capacitor v_c will lag the voltage *E*. Therefore, we obtain two sinusoidal signals with a phase difference.

10.4.2 Phase Difference Calculation using Lissajous Figures

The phase difference between two sinusoidal signals can be determined using Lissajous figure obtained on the CRO screen by applying the two signals to two pairs of deflection plates. This method is called the *X*-*Y* phase measurement.

Now let us discuss the formation of Lissajous figure on the CRO screen. Suppose that two sinusoidal signals having the same frequency but different phases are superimposed. If the phase difference is θ, these may be written as

$$v_1 = a \sin \omega t$$

and $$v_2 = b \sin (\omega t + \theta).$$

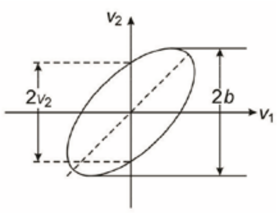


Fig. 10.6: Phase measurement using Lissajous pattern.

You may appy the voltage v_1 to the vertical deflection plate and v_2 to the horizontal deflection plate. You may recall from Unit 17 of the theory course on Mechanics, that depending on the value of phase difference, θ, the resultant pattern will be either an ellipse or a straight line. In this experiment, you have to ensure that you obtain an ellipse on the CRO screen, as shown in Fig. 10.6.

At $t = 0$, you have $v_2 = b \sin \theta$ and therefore $\sin \theta = v_2/b$ i.e. $\theta = \sin^{-1}(v_2/b)$. Note that b corresponds to the maximum value of v_2.

Fig.10.7 illustrates two possible patterns on the screen of a CRO. These figures depict possible phase difference between the two sinusoidal signals.

You have learnt in Unit 17 of the theory course on Mechanics that the ellipse present in second and fourth quadrant of the *X-Y* plane corresponds to the phase angle (π-$\sin^{-1}(v_2/b)$).

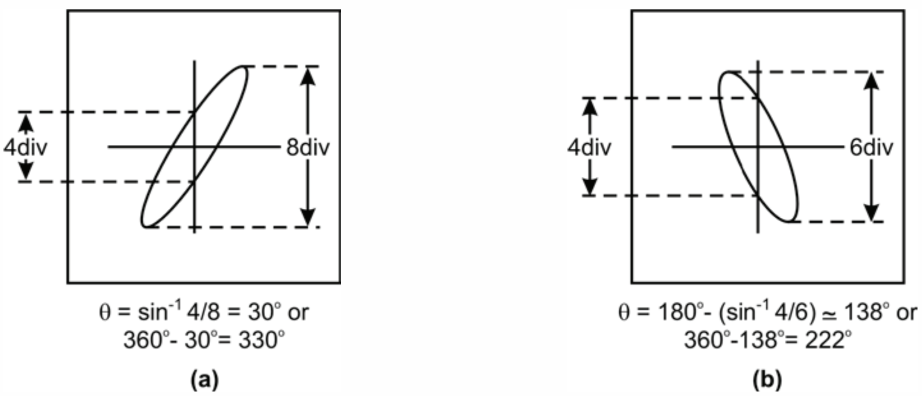


Fig. 10.7: Phase measurements.

Procedure

1. Construct the network given in Fig. 10.8 to obtain two sinusoidal signals of same frequency, with a phase difference between them.

2. Choose the *x-y* mode of CRO and apply the two signals to vertical and horizontal (*y* and x) inputs of the CRO as shown in the circuit. This will result into an elliptical figure on the CRO screen.

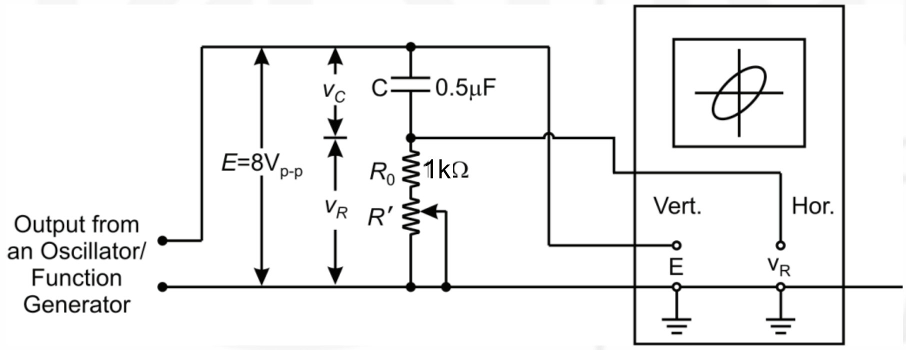


Fig. 10.8: Circuit for measurement of phase difference between two sine waves.

3. Trace the ellipse on the tracing paper and then paste it on a graph paper after carefully centring.

4. Measure $2v_2$ and $2b$ and calculate the θ value of phase as shown in Fig. 10.7.

5. Now set the value of *R* by varying *R ′* with the help of resistance box, but keeping R_0 = 1 kΩ.

6. Record the value of R', *R*, v_2 and *b* for each case in Observation Table 10.4.

7. Now calculate the value of the phase angle θ.

8. Take readings for at least 3 different values of R'.

Observation Table 10.4: Phase difference from Lissajous figures

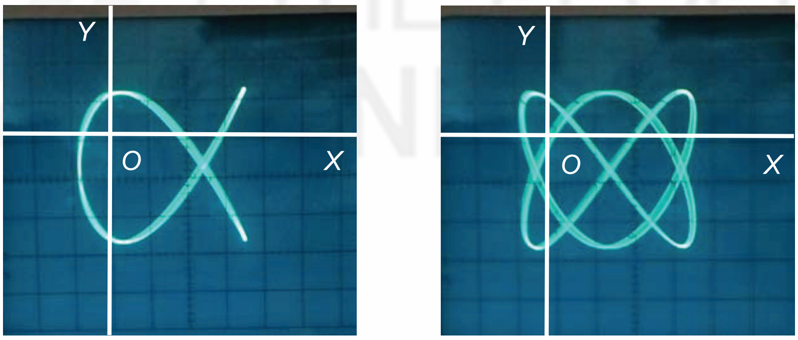
Constant resistance, R_0 = 1000 Ω

Resistance chosen in Resistance Box R' (Ω)	$R = R_0 + R'$ (Ω)	v_2	b	$\theta = \sin^{-1}(v_2/b)$
0	1000			
1000	2000			
2000	3000			

After studying the Lissajous figures for same frequency sinusoidal waves, now you will study the figures generated by two sinusoidal signals of unequal frequencies.

10.5 LISSAJOUS FIGURES OF UNEQUAL FREQUENCY SINUSOIDAL WAVES

You have learnt in the Mechanics course that the Lissajous figures can also be observed in case of two unequal frequency waves when they are applied in perpendicular direction. Let ω_x be the frequency of the sine wave given to the *x*-input and ω_y be the frequency given to the *y*-input. Let the relation between ω_x and ω_y be $m\omega_x = n\omega_y$, (where *m* and *n* are integers), then a Lissajous figure can be observed on the CRO screen such that it cuts *x*-axis at maximum *n*-times (or its multiples) and *y*-axis at maximum *m* times (or its multiple). An example of this is shown in Fig. 10.9(a and b).



(a) **(b)**

Fig. 10.9: Lissajous figures corresponding to $2\omega_x = 3\omega_y$.

In Fig. 10.9a, the Lissajous figure cuts the *x*-axis at maximum three points while the *y*-axis at maximum two points. That is in the same time period, there are three vibrations parallel to *OY*-axis and two vibrations parallel to *OX*-axis.

Hence, the frequency relation between ω_x and ω_y is $2\omega_x = 3\omega_y$. Now, in Fig. 10.9b, the figure cuts *x*-axis at maximum 6 points and *y*-axis at maximum 4 points. Again the frequency relation is $4\omega_x = 6\omega_y$ i.e. $2\omega_x = 3\omega_y$.

You can observe various patterns by changing the ratio of frequencies of sinusoidal wave given to *x*- and *y*-axis.

To perform this experiment, you will need one dual trace oscilloscope with *x-y* mode of operation and two function generators. Now perform the experiment as described in the following steps:

1. Connect the two function generator outputs to channel- I and channel- II of the CRO. Select *x-y* mode of CRO operation. Select sinusoidal waveforms on two function generators and set their frequencies.

2. Now, by adjusting the frequencies of the two function generators in various proportions you will be able to observe different Lissajous figures. Some examples of these patterns are given in Fig. 10.10.

3. Obtain the patterns on the screen and trace them on a tracing paper.

4 Write down the frequencies applied to *x* and *y* axes on this traced figure. Confirm that the *m*:*n* ratio seen on the figure matches with the corresponding pattern in Fig. 10.10.

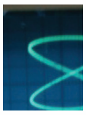
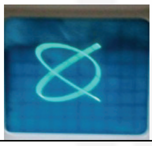
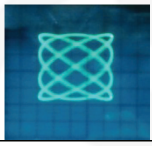
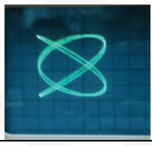
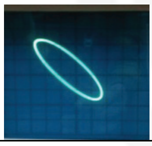
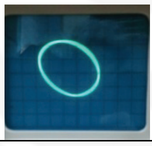
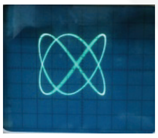
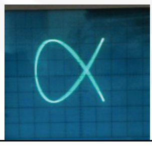
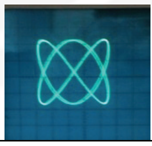
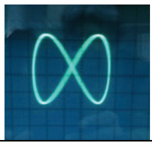
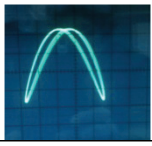
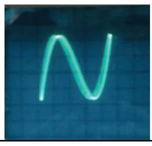
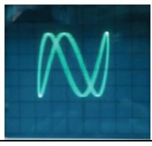
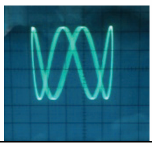
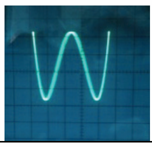
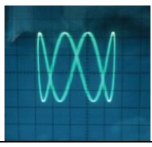
Patterns			**Frequency to x-channel**	**Frequency to y-channel**	**Ratio of $\omega_x : \omega_y$**
			4000 Hz	2000 Hz	2:1
			4000 Hz	3000 Hz	4:3
			2000 Hz	2000 Hz	1:1
			2000 Hz	3000 Hz	2:3
			2000 Hz	4000 Hz	1:2
			1000 Hz	3000 Hz	1:3
			1000 Hz	4000 Hz	1:4

Fig. 10.10: Lissajous figures corresponding to various ratios of ω_x and ω_y.

NOTE